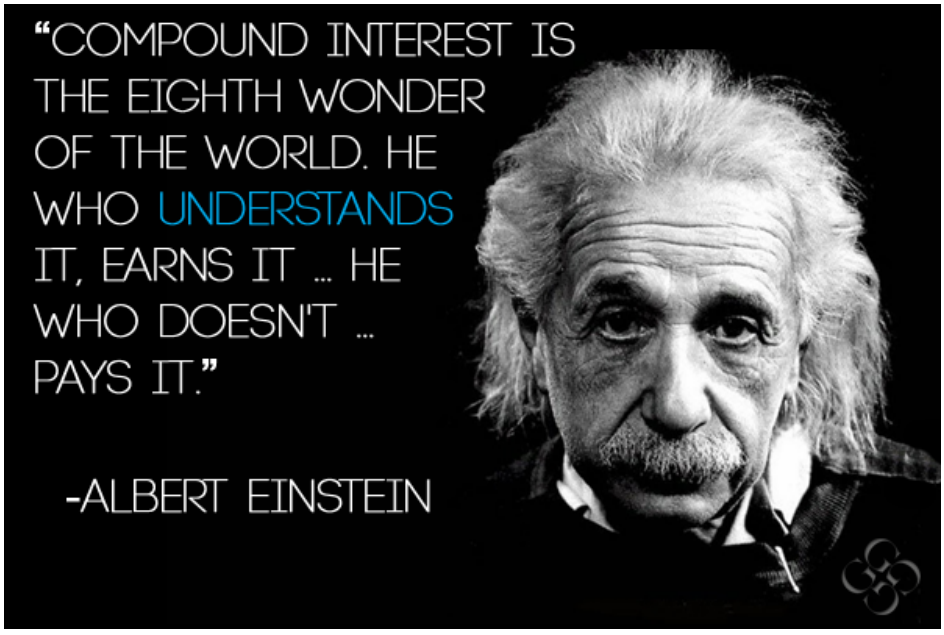
The Most Powerful Force in the Universe



You've already seen simple interest.

Eg: you invest $100 at 10% and a year later you have $110.

Easy.

Also: Not realistic and not the way the world, money, or banks actually work.

∴ after 5 years you would have $150, right?

Investment	Interest Made	Total
1000	100	1100
1100	110	1210
1210	121	1331
1331	133	1464
1464	146	1611
1611	161	1772
1772	177	1949
1949	195	2144

The idea is that we make interest on our interest.

So, our interest formula is incomplete.

$$A = P(1 + r)^t$$

A = Amount you'll have at the end
P = principle that you started with
r = rate of return (in decimal)
t = time of the investment (in years)

Should take into account the number of times per year that interest is given. For example, if ½ of the interest was given at the 6 month point then that interest would earn you interest in the second half of the year…

The formula becomes:

$$A = P(1 + \frac{r}{n})^{nt}$$

n = number of times per year that interest is given.

If n = 1, and t = 1, then we get the simple interest formula that you used in the past.

Let's invest $1,000 at 12% interest investment for 12 years.

Number of Periods (n)	Final Amount
1 $A = P(1+\frac{r}{n})^{nt}$	= 3 895.98
2 $= 1000(1+\frac{.12}{1})^{1(12)}$	= 4 044.93
3	= 4 103.93
4	= 4 132.25
10	= 4184.67
100	= 4217
1×10^{100}	=

Now let's invest $1 at 100% interest for 1 year.

Number of Periods (n)	Final Amount
1 $A = P(1+\frac{r}{n})^{nt}$	2
2 $A = 1(1+\frac{1}{1})^{1(1)}$	2.25
3	2.37
4	2.44
10	2.59
100	2.70
1×10^{100}	2.71

What happens if we allow n to grow to infinity?!
This is the foundation of calculus. Here's a sneak peek:

$$L = \lim_{n \to \infty} (1 + \frac{1}{n})^n$$

= e (2.71828...)

This is for 'continuous' interest.

$$A = pe^{rt}$$

This is how things really happen. Temperature. Growth. Decay. Continuous exponential change is modelled continuously. This gives us *e*.

Assigned Work:

1) You invest $500 at 5% compounded quarterly for 5 years. How much money will you have at the end of this investment?

2) A bank is offering an Investment that pays 6% compounded continuously. How much would a deposit of $2000 earn over 8 years?

3) You have $1,000 to invest. You are 16 years old. You plan to retire when you are 65. Assume the interest is continuous.

A) How much will you have at 3% (Savings Bond)
B) @ 6% (GIC - good one)
C) @ 12% (market average over a long period)

4) How much should you invest at 4.8% compounded continuously to have $5000 in 2½ years?

5) In 1950, the world's population was 2,555,982,611. With a growth rate of approximately 1.68%, what was the population in 1955?

6) At 5pm, you count 26,300 alien bacteria in your petri dish. If the growth rate is 2.7% per hour, how many bacteria will there be at midnight?