

Let's Get Rational

A rational expression is an algebraic fraction with a polynomial in the numerator and/or denominator.

Think of this unit as "factoring with fractions". Sound fun?

$$\frac{3}{1} \quad \frac{1}{2x} \quad \frac{x+2}{x^2-3x+4}$$



$$\frac{x^2+1}{1}$$



The additional rule that you have to remember is that you cannot divide by zero. As we will have variables in our denominators, this will be a possibility. We must show the non-permissible values.

Determine non permissible values: (where divide by zero is a possibility)

$\frac{2x}{x-2}$ <p style="text-align: center;">NPU</p> $x-2 \neq 0$ $x \neq 2$	$\frac{5}{2xy^3}$ $x \neq 0$ $y \neq 0$
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$$\frac{5x-6}{x^2-3x+2}$$

$$x^2 - 3x + 2 \neq 0$$

$$x^2 - x - 2x + 2 \neq 0$$

$$x(x-1) - 2(x-1) \neq 0$$

$$(x-1)(x-2) \neq 0$$



$$x - 1 \neq 0 \quad \text{AND} \quad x - 2 \neq 0$$

$$x \neq 1 \quad \quad \quad x \neq 2$$

Factor:

1. Greatest Common Factor
 - Always. Everytime.
2. Look for a difference of squares
 - $a^2 - b^2 = (a + b)(a - b)$
3. Quadratic Factoring (**decomposition**)
 - We will want to factor so that we can cancel. Do not use the method of graphing, or completing the square. You need to factor.

Simplify:

$$\begin{array}{c}
 x^2 + 4x + 4 \\
 \swarrow \quad \searrow \quad \downarrow \\
 x^2 + 2x + 2x + 4 \\
 \hline
 x(x + 2) + 2(x + 2) \\
 \hline
 (x + 2)(x + 2) \\
 (x + 2)^2
 \end{array}$$

$$\frac{x+2}{x^2+4x+4}$$

$$\begin{array}{l}
 m \rightarrow 4 \\
 a \rightarrow 4 \\
 (2, 2)
 \end{array}$$

$$\frac{\cancel{x+2}}{(\cancel{x+2})(x+2)} = \frac{1}{x+2}$$

(NPN)

$$\begin{array}{l}
 x+2 \neq 0 \\
 x \neq -2
 \end{array}$$

$m \rightarrow 15 \checkmark$
 $a \rightarrow 8 \checkmark$
 $(5, 3)$

$$\begin{aligned}
 & x^2 + 8x + 15 \\
 & \swarrow \quad \searrow \quad \searrow \\
 & x^2 + 5x + 3x + 15 \\
 & \underline{x(x+5) + 3(x+5)} \\
 & (x+5)(x+3)
 \end{aligned}$$

$$\frac{2x^2 + 6x}{x^2 + 8x + 15} = \frac{2x(x+3)}{(x+5)(x+3)}$$

$$= \frac{2x}{x+5}$$

\cancel{NPD}
 $x+5 \neq 0$
 $x \neq -5$
 $x+3 \neq 0$
 $x \neq -3$

\cancel{NPD}
 $x \neq \pm 4$

$$\frac{8-2x}{x^2-16}$$

$$\frac{2(4-x)}{(x+4)(x-4)}$$

$$\begin{aligned}
 & \frac{-2(-4+x)}{(x+4)(x-4)} \\
 & \frac{-2}{x+4}
 \end{aligned}$$

$$\frac{16x^2 - 9y^2}{8x - 6y}$$

$$\frac{16x^2 - 9y^2}{2(4x - 3y)}$$

$$\frac{(4x + 3y)(4x - 3y)}{2(4x - 3y)}$$

$$\frac{4x + 3y}{2}$$

$$\begin{aligned} 2(4x - 3y) &\neq 0 \\ 4x - 3y &\neq 0 \\ \checkmark 4x &\neq 3y \end{aligned}$$