02 Difference of Squares %2F Cubes

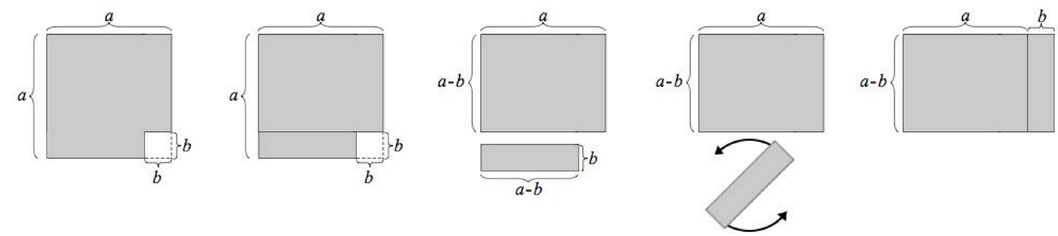
Thursday, September 20, 2018 8:41 AM

02
Difference...

Squares and Cubes

There are a couple of tricks to help you solve quickly. We must memorize them so that we can identify these types if situations when we come across them.

The first is a difference of squares. $(a^2 - b^2) = (a+b)(a-b)$



Remember how to multiply binomials?

$$(x+6)(x-6)$$

$= x^2 - 6x + 6x - 36$

$= x^2 - 36$

A difference of squares is just going the other way.

$a = x \quad b = 6$

$$x^2 - 36$$

$= (x+6)(x-6)$

$(a^2 - b^2) = (a+b)(a-b)$

$a^2 + b^2 =$

Lets Try:

$x^2 - 4$	$4x^2 - 9$	$16x^2 - 25y^2$
$(x+2)(x-2)$	$(2x+3)(2x-3)$	$(4x+5y)(4x-5y)$

Sometimes it won't be obvious. You will have to factor before you can get to it.

Eg: $3(x-y)^2 - 12$

$\rightarrow 3((x-y)^2 - 4)$

$3[(x-y)+2][(x-y)-2]$

$(a+b)(a-b)$

$a = (x-y)$

$b = 2$

$3+6$

$3(1+2)$

Here's another:

$2x^3y - 8xy^3$

$= 2xy[x^2 - 4y^2]$

$= 2xy(x+2y)(x-2y)$

$a = x \quad b = 2y$

One more tricky example:

$9(x-1)^2 - 16(x+2)^2$

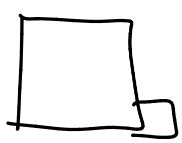
Practice:

$x^2 - 64$ $(x+8)(x-8)$	$(x-1)^2 - 196$ $((x-1)+14)(x-1-14)$
$25 - x^2$	$x^2 - 144$
$9x^2 - 121$	$x^2 - 81$
$(x-2x)^2 - 16$	$324x^2 - 289$
$x^2 - 4y^2$	$81 - x^2$

The most common mistake that students make with this is that they want to use a difference of squares factoring when they have a sum of squares in the question.

$$x^2 + 36$$
$$= x^2 + 36$$

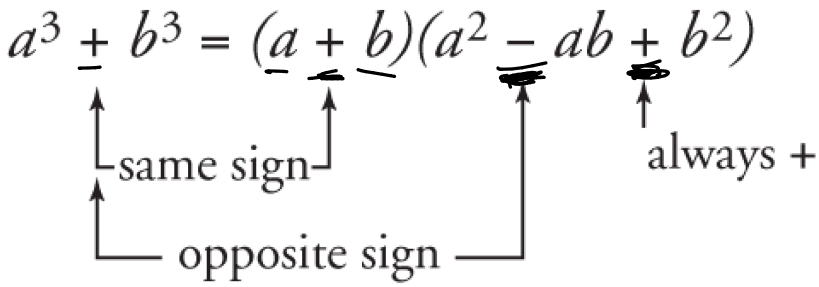
$$\sqrt{x^2 + 36} \neq \sqrt{x} + \sqrt{36}$$



$$2^2 + 3^2$$

$$(2+3)(-3)$$

Difference / Sum of cubes:



$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

If you have two cubes, either a sum or a difference, you can factor with the above formula.

$a = 2x \quad b = 3$

$$8x^3 - 27$$

$(a - b)(a^2 + ab + b^2)$

$(2x - 3)(4x^2 + 6x + 9)$

$$x^3 + 64y^3$$

$a = x \quad b = (4y)$

$(a + b)(a^2 - ab + b^2)$

$(x + 4y)(x^2 - 4xy + 16 \quad)$

Just like with difference of squares, sometimes we'll need to factor first: $2x^3 + 128y^3$

$2(x^3 + 64y^3)$

$2(x + 4y)(x^2 - 4xy + 16y^2)$

Here's a fantastic example that just never seems to end! Expect something like this on your test!

$$x^6 - y^6 = \underbrace{\left(x^3\right)^2 - \left(y^3\right)^2}_{\text{difference of squares}}$$

$$= \underbrace{\left(x^3 + y^3\right)}_{\text{sum of cubes}}\underbrace{\left(x^3 - y^3\right)}_{\text{difference of cubes}}$$

$$= \left[(x+y)\left(x^2 - xy + y^2\right)\right]\left[(x-y)\left(x^2 + xy + y^2\right)\right]$$

$$= (x+y)\left(x^2 - xy + y^2\right)(x-y)\left(x^2 + xy + y^2\right)$$

You Try:

$x^3 + 125$	$125x^3 - 27$
$-54x^3 + 16y^3$	$64x^6 - y^6$

Each of the following expressions is either a difference of perfect cubes or a sum of perfect cubes. Factor appropriately.

1) $a^3 + 1$

2) $a^3 - 1$

3) $x^3 - 8$

4) $m^3 + 8$

5) $p^3 + q^3$

6) $k^3 - h^3$

7) $y^3 - 8x^3$

8) $8p^3 + q^3$

9) $64p^3 + n^3$

10) $27x^3 - 1$

11) $27a^3 - 64b^3$

12) $125t^3 + 8s^3$

13) $27r^3 + 1000s^3$

14) $216z^3 - w^3$

15) $125m^3 - 8p^3$

16) $m^6 - 8$

17) $64y^6 + 1$

18) $8k^6 - 27q^3$

19) $125z^3 + 64r^6$

20) $(a - b)^3 - (a + b)^3$