

# Conservation of Energy

The total energy of the universe is the same today as it was yesterday. This will be the same tomorrow... and the next day.

The total amount of energy before an event is exactly the same as after the event. Always. Forever. Absolutely.

Energy can change forms:

$$E = mc^2$$

But, the total amount does not change.

Last class we talked about:

$$E_p = mgh$$

$$E_k = \frac{mv^2}{2} = \frac{1}{2}mv^2$$

Let's use these two energies in the Law of Conservation of Energy:

*Total Energy Before = Total Energy After*

$$E_{p0} + E_{k0} = E_{pf} + E_{kf} + E_H$$

Before      after + heat

↳ loss  
↳ inefficiency.

Alexander weighs 65kg. He climbs a tree (because that's an awesome thing to do, and he's an awesome guy) to a height of 2.5m.

How much potential energy does Alex have?

$$\begin{aligned} E_p &= mgh \\ &= 65(9.81)(2.5) \\ &= 1.6 \text{ kJ} \end{aligned}$$

How much kinetic energy will Alex have when he hits the ground?

$$E_k = 1.6 \text{ kJ}$$

$$E_k < 1.6 \text{ kJ}$$

With the **law of conservation of energy**, we can know about what the future energy state of an object will be.

This is at Cedar Point. Let's say the first hill is 25m up. The second hill is 15m up. The lowest point is 5m off of the ground.



What will the speed of the cart be at the top of the second hill?

$$\begin{aligned} E_p &= mgh & = E_p + E_k \\ &= m(9.81)(20) & = m(9.81)(0) + \frac{mv^2}{2} \\ &= 196 \text{ m} & \end{aligned}$$

$$196m = 98.1m + \frac{mv^2}{2}$$

Do we need the mass?

$$196 = 98.1 + \frac{v^2}{2}$$

$$\sqrt{2(196 - 98.1)} = v$$

$$14 \frac{m}{s} = v$$

Let's see how this new knowledge compares to kinematics!

Kenzie drops a 5kg rock off the top of a 15m building to land on a zombie below. What will the rock's velocity be when it is 5.0m above the ground?

$$\left( E_p + E_k \right)_{\text{before}} = \left( E_p + E_k \right)_{\text{after}}$$

$$m(9.81)(15) + 0 = m(9.81)(5) + \frac{mv^2}{2}$$

$$\sqrt{2(9.81)(15) - (9.81)(5)} = v$$

$$14 \frac{m}{s} = v$$

However, Sydney had a speed detector and found that the rock only had a velocity of 13.2m/s. Where did the energy go?

$$E_p = E_k + E_p + E_H$$

$$5(9.81)(15) = \frac{5(13.2)^2}{2} + 5(9.81)(5) + E_H$$

$$54.155 = E_H$$

$$54.2 = E_H$$

To celebrate the death of this zombie, Nic throws his hat in the air. If he throws with an initial velocity 20.0m/s straight up, how high will the hat go? (neglect air resistance)

$$E_{p0} + E_{k0} = E_{pf} + E_{kf}$$

$$mgh_0 + \frac{mv_0^2}{2} = mgh_f + \frac{mv_f^2}{2}$$

$$gh_0 + \frac{v_0^2}{2} = gh_f + \frac{v_f^2}{2}$$

$$gh_0 - gh_f = \frac{v_f^2 - v_0^2}{2}$$

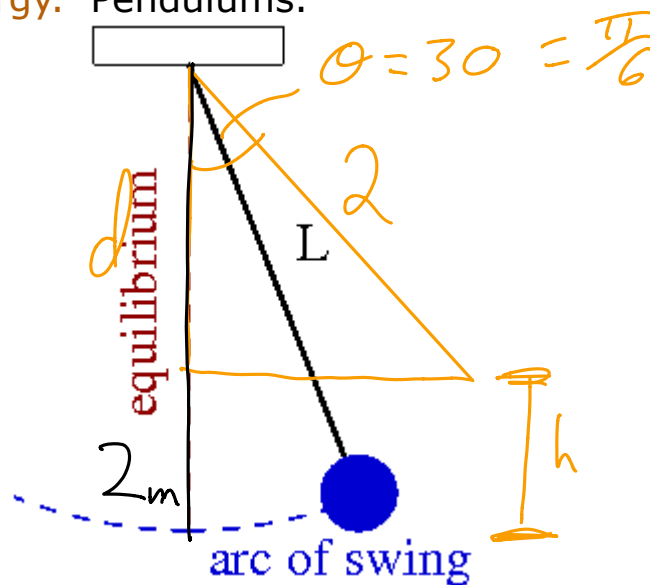
$$2g(h_0 - h_f) = v_f^2 - v_0^2$$

$$v_0^2 + 2g(h_0 - h_f) = v_f^2$$

One more mind blowing step! Who sees it?

$$v_f^2 = v_0^2 + 2ad$$

One more type of question to be solved with the law of conservation of energy. Pendulums.



If the length of the string is 2.0m, and the bob is  $30^\circ$  above equilibrium:

How high is the bob from the bottom?

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos(30) = \frac{d}{2}$$

$$d = \sqrt{3} \approx 1.73$$

$$h = 2 - 1.73 = 0.27 \text{ m} = 27 \text{ cm.}$$

What velocity will the bob have at this point?

The diagram shows a pendulum bob at an angle from the vertical equilibrium position. A vertical dashed line represents the equilibrium position. The height of the bob above the lowest point is labeled 'h'.

$$v_f^2 = v_o^2 + 2ad$$