Factoring Quadratics

What it is and why we do it

Factoring is taking an equation that has terms being added and subtracted and changing it into an expression that only has terms multiplied.

$$2x^2+x=0 \rightarrow x(2x+1)=0$$

Why do we do it? **Why** bother? **What** is the advantage?

If we have one number plus another equals zero, there are literally an infinite number of solutions for this problem.

$$x+y=0$$

If x=0	1	2	3	4	5	...
Then y=0	-1	-2	-3	-4	-5	-...

However: if those numbers are multiplied, at least one of those numbers MUST be zero!

$$xy=0$$

If x is not zero, what value can y have that would cause the expression to be valid? y must be zero! One or both of x,y MUST be zero.

We factor equations to remove the adding and subtracting terms. Once we have EVERYTHING only multiplied AND it is set equal to zero, then we can say that at least one of those multiplied pieces is zero.

$x^2+1=0$
$x=\pm\sqrt{-1}$

$x^2-1=0$
$x^2=1$
$x=\pm 1$

$(x^2+1)(2x+4)(x^2-1)(5)=0$

$2x+4=0$
$x=\frac{-4}{2}$
$x=-2$

$x=\pm 1, \pm i, -2$

Hint: if you only have 2 terms, see if you have a difference of squares.

$$a^2 - b^2 = (a+b)(a-b)$$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+4=0$ or $x-4=0$

$x=-4$ $\quad$ $x=4$

$x=\pm 4$

If you have 4 terms, look and see if any of those terms can be grouped together.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$2x^2 + 3x + 5 = 6$$

$2x^2+3x-1=0$

$m \to -2$
$a \to 3$

$2, 1$

If you have 3 terms, we use decomposition.

Examples: Solve ~~Factor~~ Completely:

$$x^2 - 8x + 15 = 0$$

m → 15
a → −8

$x^2 - 5x - 3x + 15 = 0$

$x(x-5) - 3(x-5) = 0$

$(-5, -3)$

$(x-5)(x-3) = 0$

$x - 5 = 0$
$x = 5$
or
$x - 3 = 0$
$x = 3$

$x = 3, 5$

$$x^2 + 9x - 22 = 0$$

$(x+11)(x-2) = 0$

$x + 11 = 0$ or $x - 2 = 0$
$x = -11$ $\quad x = 2$

$x = 2, -11$

$$3x^2 + 3x - 6 = 0$$

m −18
a 3

$(6, -3)$

$3x^2 - 3x + 6x - 6 = 0$

$3x(x-1) + 6(x-1) = 0$

$(x-1)(3x+6) = 0$

$x - 1 = 0$ or $3x + 6 = 0$
$x = 1$ $\quad x = -\frac{6}{3} = -2$

$x = 1, -2$

$$\frac{1}{2}x^2 - x - 4 = 0$$

2

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

Examples (difference of squares):

$$x^2 - 9 = 0$$

$$(x+3)(x-3) = 0$$

$$x+3 = 0 \quad \text{or} \quad x-3=0$$

$$x = -3 \qquad x = 3$$

$$x = \pm 3$$

$$-64 + x^2 = 0$$

$$x^2 - 64 = 0$$

$$(x+8)(x-8)$$

$$x = \pm 8$$

$$2x^2 - 50 = 0$$

$$2(x^2 - 25) = 0$$

$$2(x+5)(x-5) = 0$$

$$x = \pm 5$$

$$x^4 - 16 = 0$$

$$(\underline{x^2 - 4})(x^2+4) = 0$$

$$(x+2)(x-2)(x^2+4) = 0$$

$$x = \pm 2, \pm 4i$$

look away high school students.

Now let's factor when the coefficient is not equal to one (and it is not easy to make it so).

$3, -4$

$$2x^2 - x - 6 = 0$$

$2x^2 - 4x + 3x - 6 = 0$

$2x(x-2) + 3(x-2) = 0$

$(x-2)(2x+3) = 0$

$x = 2, -\frac{3}{2}$

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12

$$4x^2 + 12x + 9 = 0$$

$4x^2 + 6x + 6x + 9 = 0$

$2x(2x+3) + 3(2x+3) = 0$ $(6, 6)$

$(2x+3)(2x+3) = 0$

$(2x+3)^2 = 0$

$x = -\frac{3}{2}$

$(-10, -1)$

$10x^2 - 22x + 4 = 0$

$5x^2 - 11x + 2 = 0$

$5x^2 - x - 10x + 2 = 0$

$x(5x-1) - 2(5x-1) = 0$

$(5x-1)(x-2) = 0$

$x = 2, \frac{1}{5}$

$12(x+2)^2 + 24(x+2) + 9 = 0$

$4(x+2)^2 + 8(x+2) + 3 = 0$

Let $x+2 = R$

$4R^2 + 8R + 3 = 0$

$4R^2 + 6R + 2R + 3 = 0$

$2R(2R+3) + (2R+3) = 0$

$(2R+3)(2R+1) = 0$

$(2(x+2)+3)(2(x+2)+1) = 0$

$(2x+7)(2x+5) = 0$

$x = \frac{-7}{2}, \frac{-5}{2}$

H/W: Pg229:
#1,2ab,3,4,5ab,6a