Multiplying and Dividing Rationals

1. We always have to state the restrictions.
   - ➢ Dividing by zero is never allowed.
   - ➢ Sometimes this can be hard to see. For example, $y = x^2 + x - 6$ does not immediately look like it has zeroes of -3 and 2.
2. Factor all numerators and denominators.
   - ➢ This is so you can cancel terms. This will often be the case.
3. State the answer as one rational.

Let's start with multiplication:

$$\frac{x^2+x-6}{x^2+2x-15} \bullet \frac{x-3}{x-2}$$

$m \to -6$, $q \to 1$ $(+3, -2)$

$x^2 + x - 6$

$x^2 - 2x + 3x - 6$

$x(x-2) + 3(x-2)$

$(x-2)(x+3)$

$x = 2$ or -3

$x - 2 = 0$ or $x + 3 = 0$

$x = 2$ $\quad x = -3$

$x^2 + 2x - 15$

$x^2 - 3x + 5x - 15$

$x(x-3) + 5(x-3)$

$(x-3)(x+5)$

$m \to -15$, $q \to 2$

$(-3, 5)$

NPV

$x \neq 3, -5, 2$

$$\frac{(x-2)(x+3)(x-3)}{(x-3)(x+5)(x-2)}$$

$$= \frac{x+3}{x+5}$$

$$\frac{x^2+7x+12}{x^2+2x-15} \cdot \frac{x^2-5x+6}{x^2-16}$$

$x^2+7x+12$ | x^2-5x+6 | $x^2+2x-15$ | x^2-16

$x^2+4x+3x+12$ | $x^2-3x-2x+6$ | $x^2-3x+5x-15$ | $(x+4)(x-4)$

$x(x+4)+3(x+4)$ | $x(x-3)-2(x-3)$ | $x(x-3)+5(x-3)$

$(x+4)(x+3)$ | $(x-3)(x-2)$ | $(x-3)(x+5)$

NPV $x \neq 3, -5, \pm 4$

$$\frac{(x+4)(x+3)(x-3)(x-2)}{(x-3)(x+5)(x+4)(x-4)}$$

$$= \frac{(x+3)(x-2)}{(x+5)(x-4)}$$

We love quadratics, we love factoring and we love fractions!

This should be the best chapter ever!

When we divide a couple things need to be remembered.

1. We invert and multiply when we have a fraction divided by a fraction.
   - ➢ This is sometimes called "multiply by the inverse".
2. You can not cancel terms until this is done.
3. We will end up with one more non permissible term.

$\frac{3}{2} \div \frac{1}{2}$ $\quad \frac{3}{2}\left(\frac{2}{1}\right)$

$$\frac{x^2-x-20}{x^2-6x} \div \frac{x^2+9x+20}{x^2-12x+36}$$

x^2-x-20

$x^2-5x+4x-20$

$x(x-5)+4(x-5)$

$(x-5)(x+4)$

$x^2+9x+20$

$x^2+5x+4x+20$

$x(x+5)+4(x+5)$

$(x+5)(x+4)$

x^2-6x

$x(x-6)$

$x^2-12x+36$

$x^2-6x-6x+36$

$x(x-6)-6(x-6)$

$(x-6)(x-6)$

$(x-6)^2$

$$\frac{(x-5)(x+4)(x-6)^2}{x(x-6)(x+5)(x+4)}$$

NPV

$x \neq 0, -5, -4, 6$

$$= \frac{(x-5)(x-6)}{x(x+5)}$$

$$\frac{x^2+15x+56}{x^2-3x-54} \div \frac{x^2+6x-16}{x^2+4x-12}$$

$x^2+15x+56$

$x^2+8x+7x+56$

$x(x+8)+7(x+8)$

$(x+8)(x+7)$

$x^2-3x-54$

$x^2-9x+6x-54$

$x(x-9)+6(x-9)$

$(x-9)(x+6)$

$x^2+6x-16$

$x^2-2x+8x-16$

$x(x-2)+8(x-2)$

$(x-2)(x+8)$

$x^2+4x-12$

$x^2-2x+6x-12$

$x(x-2)+6(x-2)$

$(x-2)(x+6)$

$$\frac{(x+8)(x+7)(x-2)(x+6)}{(x-9)(x+6)(x-2)(x+8)}$$

NPV: $-6, 9, 21, -8$

$x \neq$

$$= \frac{x+7}{x-9}$$

Homework: pg: 327

#1,2,4,7,8abc,10,15,16