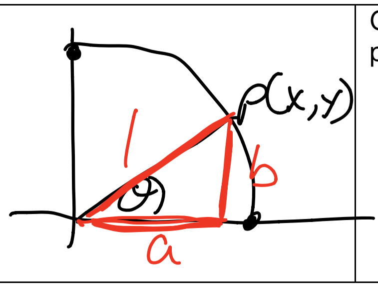
6.2 Using the Pythagorean Identities



Given a point *P* (*x*, *y*) on a terminal arm of an angle θ, in standard position.

$a^2+b^2=c^2$ SohCahToa

$\sin\theta = \frac{y}{1}$ $\cos\theta = \frac{x}{1}$

$\cos^2\theta + \sin^2\theta = 1$

Other Form (using tangent)	Other form (using co-tangent)
$\frac{\sin^2\theta}{\cos^2} + \frac{\cos^2\theta}{\cos^2} = \frac{1}{\cos^2}$ $\tan^2\theta + 1 = \sec^2\theta$	$\frac{\sin^2\theta}{\sin^2} + \frac{\cos^2\theta}{\sin^2} = \frac{1}{\sin^2}$ $1 + \cot^2\theta = \csc^2\theta$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$	$1+\tan^2\theta = \sec^2\theta$	$1+\cot^2\theta = \csc^2\theta$

Other Forms (Not provided… but useful)

$\cos^2\theta = 1-\sin^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$\cot^2\theta = \csc^2\theta - 1$
$\sin^2\theta = 1-\cos^2\theta$	$1 = \sec^2\theta - \tan^2\theta$	$1 = \csc^2\theta - \cot^2\theta$

A note about notation, we've agreed that: $\sin^2 x = (\sin x)^2$ to save ink (really :S)

Using the previous identities, prove:

$\sin^2\theta\cot^2\theta = 1-\sin^2\theta$	$\frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$
RHS: $1-\sin^2\theta$ $= \cancel{\sin^2}\frac{\cos^2}{\cancel{\sin^2}}$ $= \cos^2$ $= 1-\sin^2\theta$	RHS: $2\sec\theta$ $\frac{\cos(1-\sin)+\cos(1+\sin)}{1-\sin^2}$ $= \frac{\cancel{\cos}[1-\cancel{\sin}+1+\cancel{\sin}]}{\cos^{\cancel{2}}}$ $= \frac{2}{\cos}$ $= 2\sec\theta$

$(1-\cos^2\theta)(1+\tan^2\theta)=\tan^2\theta$ $1+\frac{\sin^2}{\cos^2}-\cos^2-\sin^2$ $1+\tan^2-(\cos^2+\sin^2)$ $1+\tan^2-1$ $\tan^2\theta$	$\sec^2\theta+\csc^2\theta=(\tan\theta+\cot\theta)^2$ A RHS $=\tan^2+\cot^2+2\tan\cot$ $=\sec^2-1+\csc^2-1+2$ $=\sec^2+\csc^2$
LHS $\frac{1-\cos x}{\sin x}=\boxed{\frac{\sin x}{1+\cos x}}$ $=\frac{(1-\cos)}{\sin}\cdot\frac{(1+\cos)}{(1+\cos)}$ $=\frac{1+\cancel{\cos}-\cancel{\cos}-\cos^2}{\sin+\sin\cos}$ $=\frac{1-\cos^2}{\sin(1+\cos)}$ $=\frac{\sin^{\cancel{2}}}{\cancel{\sin}(1+\cos)}$ $=\frac{\sin}{1+\cos}$	$\frac{\sin^3 x}{1+\cos x}=\boxed{\frac{\tan x(1-\cos x)}{\sec x}}$ A $\left(\frac{1-\cos}{1-\cos}\right)\frac{\sin^3}{1+\cos}$ = $\frac{\sin^3-\sin^3\cos}{1+\cos-\cos-\cos^2}$ $\frac{\sin^3(1-\cos)}{1-\cos^2}$ = $\frac{\sin^3(1-\cos)}{\sin^2}$ = $\sin(1-\cos)$ = $\cancel{\sin}\,\frac{\cos}{\cos}(1-\cos)$ $\boxed{\frac{\tan(1-\cos)}{\sec}}$ $\frac{\sin\cos(1-\cos)}{\cos}$ $=\sin(1-\cos)$