

### 6.4 Double Angle Identities

Identity:  $\sin 2\theta = 2 \sin \theta \cos \theta$

Simplified Form	Expanded Form
$\sin(2 \times 5)$	$2 \sin(5) \cos(5)$
$\sin(2(34))$ $= \sin(68)$	$2 \sin 34 \cos 34$
$3 \sin(2(7x))$ $= 3 \sin(14x)$	$6 \sin 7x \cos 7x$ 3-2
$7 \sin 4x$ $7(\sin[2(2x)])$	$7(2 \sin(2x) \cos(2x))$ $14 \sin(2x) \cos(2x)$

Practice Questions:

Simplify: $10 \sin 7x \cos 7x$	Expand: $20 \sin 6x$
a. $20 \sin 14x$ b. $5 \sin 3.5x$ c. $5 \sin 14x$ d. $20 \sin 3.5x$	a. $10 \sin 12x \cos 12x$ b. $40 \sin 3x \cos 3x$ c. $40 \sin 12x \cos 12x$ d. $120 \sin x \cos x$

Cross-Topic Question:

What is the period and amplitude of  $y = 10 \sin 2x \cos 2x$

	Amp	Period
a.	10	$\pi$
b.	5	$\frac{\pi}{2}$
c.	20	$2\pi$
d.	5	$\pi$

$$y = 5 \sin(2(2x))$$

$$= 5 \sin 4x$$

$$P = \frac{2\pi}{4}$$

$$= \frac{\pi}{2}$$

Identity:  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{\sin 2\theta}{\cos 2\theta}$

Simplified Form	Expanded Form
$\tan 36$ $\tan(2 \cdot 18)$	$\frac{2 \tan(18)}{1 - \tan^2(18)}$
$\tan 10x$ $\tan(2 \cdot 5x)$	$\frac{2 \tan(5x)}{1 - \tan^2(5x)}$
$3 \tan(18x)$ $3 \tan(9 \cdot 2x)$	$\frac{6 \tan(9x)}{1 - \tan^2(9x)}$
$\tan(6x)$	$\frac{2 \tan(3x)}{1 - \tan^2(3x)}$
$3 \tan(16x)$	$\frac{6 \tan(8x)}{1 - \tan^2(8x)}$

$$= \cos^2 - (1 - \cos^2)$$

$$1 - \sin^2 - \sin^2$$

Identities:

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

Simplified Form	Expanded Form
cos 14	
$\cos(2 \cdot 7)$	$\cos^2(7) - \sin^2(7)$
cos 14	
	$2\cos^2(7) - 1$
cos 14	
	$1 - 2\sin^2(7)$
$\cos(14x)$	$\cos^2 7x - \sin^2 7x$
$\cos(2 \cdot 3x)$	$2\cos^2(3x) - 1$
$2[\cos(2 \cdot 5x)]$	$4\cos^2 5x - 2$ $2(2\cos^2 5x - 1)$
$\cos(2 \cdot 5x)$	$1 - 2\sin^2(5x)$
$\cos(2 \cdot 8x)$	$5 - 10\sin^2 8x$ $5(1 - 2\sin^2 8x)$

Practice Questions:

Simplify:  $3 - 6\sin^2 10x$

- $\cos 20x$
- $3\cos 20x$
- $6\cos 5x$
- $1.5\sin 20x$

Expand:  $8\cos 14x$

- $8\cos^2 7x - 1$
- $16\cos^2 7x - 1$
- $16\cos^2 7x - 8$
- $4\cos^2 28x - 2$