Slopes & Area "Oh my!"

Let's look at some of our kinematic equations and see how they relate to slope and area:

final − initial

$$\Delta d = v\Delta t \;||\; \Delta v = a\Delta t$$

Show these as a slope: (remember $m = \frac{y_2 - y_1}{x_2 - x_1}$)

$$v = \frac{d_f - d_i}{t_f - t_i} \quad || \quad a = \frac{v_f - v_i}{t_f - t_i}$$

Remember how to find $v_{average}$? (constant velocity)

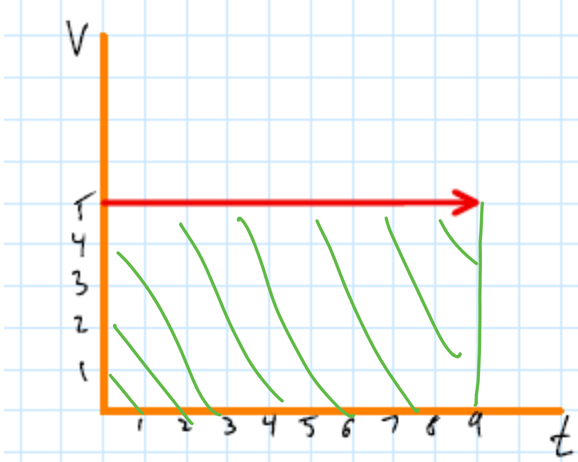
$$v_{avg} = \frac{v_f + v_0}{2}$$

$$\frac{100 + 100}{2} = \frac{200}{2} = 100$$

Use this in our first equation:

$$\Delta d = \frac{v_f + v_0}{2}\Delta t$$

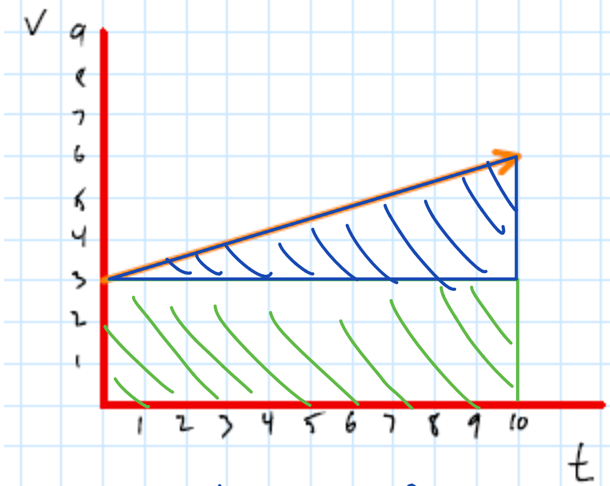
$$\frac{100 + 0}{2} = 50$$



What is the area under the line?

Remember d= vt

$45_m = 5\frac{m}{\cancel{s}}(9\cancel{s})$



What is the area under the line?

I have two shapes. One triangle,
And one rectangle.

What is this area? Remember d=vt

$d = A_{rec} + A_{\Delta}$

$= V_o t + \frac{(v_f - v_i)t}{2}$

$10(3) + \frac{3(10)}{2}$

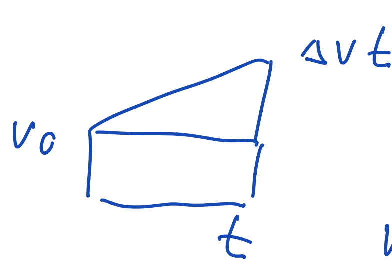
$30 + 15$

> 45

Area of a triangle $\frac{bh}{2}$ in this case $\frac{vt}{2}$. We also know that $\Delta v = at$

$$\therefore \frac{[at]t}{2} = \frac{at^2}{2}$$

We put the area of the rectangle together with the angle of the triangle and we find that:



$\square + \Delta$

$V_o t + Vt$

$$\overline{\ \ 2\ \ }$$

$$V = \underline{at}$$

$$\boxed{\Delta d = v_0 t + \frac{at^2}{2}}$$

Pretty neat, eh?

Let's do an example:

An airplane accelerates down a runway at 3.20m/s² for 32.8 seconds until it finally lifts off the ground. Determine the distance travelled before takeoff.

Step 1: Write down what we know, and what we need to know.

$a = 3.2 \quad t = 32.8 \quad d = ?$

$$d = V_0 t + \frac{at^2}{2}$$

$V_f = ?$

$$d = 0(32.8) + \frac{(3.2)(32.8)^2}{2}$$

$d = 1.72 \times 10^3 m$ or $1.72 km$.

2: A particular airport runway is 150m. One kind of airplane that might use this airfield can accelerate at 2.0m/s².

A) What speed can this airplane reach if it accelerates fully down the runway?

B) The plane needs a velocity of 27.8 m/s to take off. Does it reach this speed?

$a = 2 \quad d = 150 \quad V_0 = 0 \quad v_f = ?$

Note: We are not trying to find time, and it is not given. But both v=at and d=vt rely on time. Let's use that formula we have not used yet!

A) $V_f^2 = V_0^2 + 2ad$

$V_f = \sqrt{0 + 2(2)(150)}$

$V_f = \sqrt{600}$

$V_f = \sqrt{100} \cdot \sqrt{6}$

$V_f = 10\sqrt{6} \ m/s$

$V_f = 24.5 \ m/s$

$$v_f^{\,2} = v_i^{\,2} + 2ad$$

But, where did that formula come from? Not exactly intuitive…

We start from the two formulas we use all the time:

$$d = vt \ || \ v = at$$

Our velocity is not constant so we use the average velocity formula…

$$d = \frac{v_f + v_0}{2} t \ || \ v_f - v_0 = at$$

Solve them both for t, and substitute:

$$\frac{2d}{V_f + V_0} = t \qquad \frac{V_f - V_0}{a} = t$$

$$t = t$$

$$\frac{2d}{V_f + V_0} = \frac{V_f - V_0}{a}$$

$$2da = (V_f - V_0)(V_f + V_0)$$

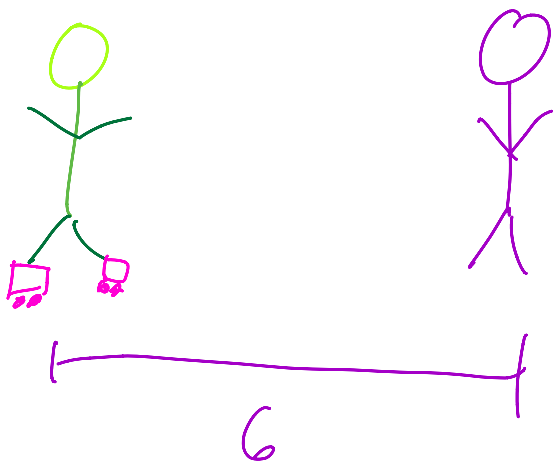
$2ad = V_f^2 + \cancel{V_f V_0} - \cancel{V_f V_0} - V_0^2$

$V_0^2 + 2ad = V_f^2$

3) Sander is rollerblading down the hallway when he sees me come around the corner. He quickly puts on the brakes in an attempt to avoid the collision. If his starting velocity is 4.5m/s and it takes him 2.5s to stop, will he hit me if I am standing 6m in front of him?

Hint1: Velocity is not constant. What is the formula for v_{avg}?

Hint2: Acceleration is the change in v over change in t.



$V_0 = 4.5\ m/s$

$t = 2.5\ s$

$d = ?$

$V_f = 0$

$d = vt$

$d = \frac{v_f + v_i}{2} t$

$d = \frac{0 + 4.5}{2} (2.5)$

$d = 5.63\ m.$

Test Practice:

1. A car starts from rest and accelerates uniformly over a time of 5.21 seconds for a distance of 110 m. Determine the acceleration of the car.

2. A race car accelerates uniformly from 18.5 m/s to 46.1 m/s in 2.47 seconds. Determine the acceleration of the car and the distance traveled.

3. Rocket-powered sleds are used to test the human response to acceleration. If a rocket-powered sled is accelerated to a speed of 444 m/s in 1.80 seconds, then what is the acceleration and what is the distance which the sled travels?

4. A bike accelerates uniformly from rest to a speed of 7.10 m/s over a distance of 35.4 m. Determine the acceleration of the bike.

5. An engineer is designing the runway for an airport. Of the planes which will use the airport, the lowest acceleration rate is likely to be 3.00 m/s^2. The takeoff speed for this plane will be 65 m/s. Assuming this minimum acceleration, what is the minimum allowed length for the runway?

6. A car traveling at 22.4 m/s skids to a stop in 2.55 s. Determine the skidding distance of the car (assume uniform acceleration).

7. A bullet leaves a rifle with a muzzle velocity of 521 m/s. While accelerating through the barrel of the rifle, the bullet moves a distance of 0.840 m. Determine the acceleration of the bullet (assume a uniform acceleration).

8. A bullet is moving at a speed of 367 m/s when it embeds into a lump of moist clay. The bullet penetrates for a distance of 0.0621 m. Determine the acceleration of the bullet while moving into the clay. (Assume a uniform acceleration.)

9. It was once recorded that a Jaguar left skid marks which were 290.0 m in length. Assuming that the Jaguar skidded to a stop with a constant acceleration of -3.90 m/s^2, determine the speed of the Jaguar before it began to skid.

10. A plane has a takeoff speed of 88.3 m/s and requires 1365 m to reach that speed. Determine the acceleration of the plane and the time required to reach this speed.

11. A dragster accelerates to a speed of 112 m/s over a distance of 398 m. Determine the acceleration (assume uniform) of the dragster.

12. You are designing an airport for planes. One kind of plane that might use this airfield must come in to land at a speed of 27.8 m/s (100 km/h) or less and can accelerate at -2.00 m/s^2. If the runway is 150.0 m long, can this airplane come to a stop before reaching the end of the runway? If not, what minimum length must the runway have?

Homework Key

1. Answer: $a = 8.10 \text{ m/ s}^2$
2. Answer: $a = 11.2 \text{ m/s}^2$, $d = 79.8$ m
3. Answer: $a = 247 \text{ m/s}^2$, $d = 400$ m
4. Answer: $a = 0.712 \text{ m/s}^2$
5. Answer: $d = 704$ m
6. Answer: $d = 28.6$ m
7. Answer: $a = 1.62*10^5 \text{ m /s}^2$
8. Answer: $a = -1.08*10^6 \text{ m /s}^2$
9. Answer: $v_i = 47.6$ m /s
10. Answer: $a = 2.86 \text{ m/s}^2$, $t = 30.8$ s
11. Answer: $a = 15.8 \text{ m/s}^2$
12. Answer: No, 193 m long