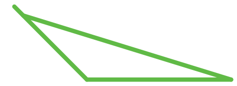
Sine Law

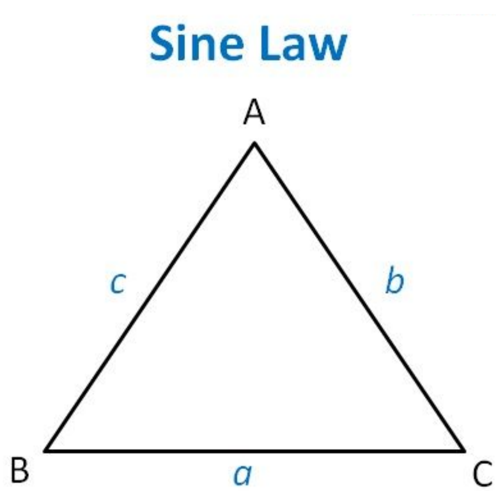


We can only use SOH CAH TOA when we are dealing with right angle trigonometry.

Many triangles we will come across will not be as favourable…

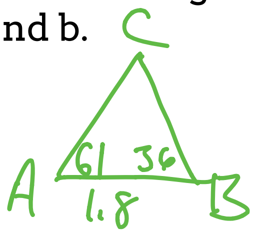
For this we have the sine law.

In essence: The ratio of the sides to the corresponding angle is the same for all 3 angle/side pairs in any triangle.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

EG: In triangle ABC, ∠A = 61°, ∠B = 36°, and side c = 1.8 km. Find side a and b.



$\angle C = 180 - 36 - 61$
$= 83°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin(61)}{a} = \frac{\sin(83)}{1.8}$$

$$a = \frac{1.8\sin(61)}{\sin 83}$$

$$\approx 1.58\text{ km}$$

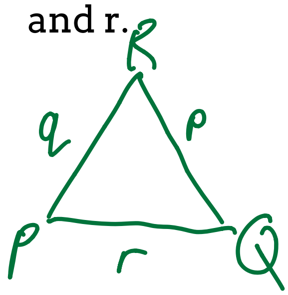
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

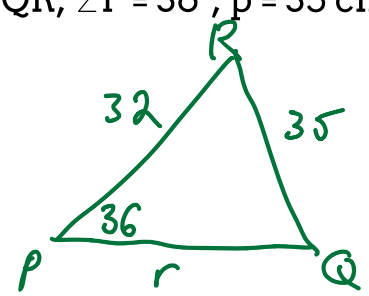
$$\frac{\sin(36)}{b} = \frac{\sin(83)}{1.8}$$

$$b = \frac{1.8\sin(36)}{\sin(83)}$$

$$\approx 1.07\text{ km}$$

EG: In triangle PQR, ∠P = 36°, p = 35 cm, and q = 32 cm. Determine ∠R and r.





$$\frac{\sin P}{p} = \frac{\sin Q}{q}$$

$$\frac{\sin 36}{35} = \frac{\sin Q}{32}$$

$$Q = \sin^{-1}\left[\frac{32\sin(36)}{35}\right]$$

$$\approx 32.5°$$

$\angle R = 180 - 36 - 32.5$
$= 111.5°$

$$①\quad \frac{\sin(111.5)}{r} = \frac{\sin(36)}{35}$$

$$②\quad r = \frac{35\sin(111.5)}{\sin(36)}$$

$$\approx 55.4\text{ cm}$$

$$\frac{\sin(111.5)}{r} = \frac{\sin(36)}{35}$$

$$35\sin(111.5) = r\sin(36)$$

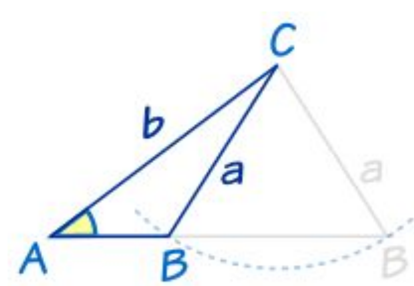
$$\frac{35\sin(111.5)}{\sin(36)} = r$$

The ambiguous case:

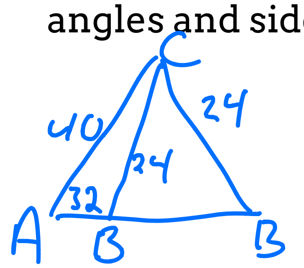
If you are given 2 angles and 1 side, then the triangle you solve for is uniquely defined. (AAS → **A**ngle **A**ngle **S**ide)

We must be aware of the ambiguous case though:

If you are given Angle then 2 sides (ASS), we have 2 possible triangles that can be formed.



EG: In $\triangle ABC, \angle A = 32°, a = 24, b = 40$ Solve the triangle (solve means all angles and sides).



Case 1
Big △

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 32}{24} = \frac{\sin B}{40}$$

$$B = \sin^{-1}\left[\frac{40 \sin(32)}{24}\right]$$

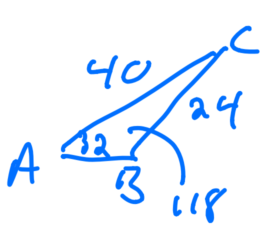
$$\doteq 62°$$

$$\angle C = 180 - 32 - 62$$
$$= 86°$$

$$\frac{\sin 32}{24} = \frac{\sin 86}{c}$$

$$c = \frac{24 \sin(86)}{\sin(32)}$$

$$= 45u$$



$$\angle C = 180 - 118 - 32$$
$$= 30°$$

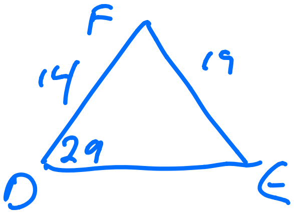
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 32}{24} = \frac{\sin 30}{c}$$

$$c = \frac{24 \sin(30)}{\sin 32}$$

$$c \doteq 22.6u$$

EG: In $\triangle DEF, \angle D = 29°, d = 19, e = 14$. Solve the triangle.



$$\frac{\sin D}{d} = \frac{\sin E}{e}$$

$$\frac{\sin(29)}{19} = \frac{\sin E}{14}$$

$$E = \sin^{-1}\left[\frac{14 \sin(29)}{19}\right]$$

$$= 20.9°$$

$$\angle F = 180 - 20.9 - 29$$
$$= 130.1°$$

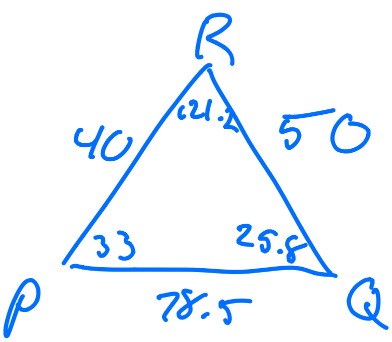
$$\frac{\sin(29)}{19} = \frac{\sin(130.1)}{f}$$

$$f = \frac{\sin(130.1)(19)}{\sin(29)}$$

$$= 30u.$$

You try:
In $\triangle PQR, \angle P = 33°, p = 5, q = 40$. Solve.

$Q = ?$ $\quad p = 50$



HINT

2.3 (For Monica that's page 108 → good morning)

1ac, 2-4bc, 5

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{b \sin A}{a} = \sin B$$

$$b \sin A = a \sin B$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{10}{5} = \frac{20}{10}$$

$$10(10) = 5(20)$$

$$\frac{10}{20} = \frac{5}{10}$$