Sine Law

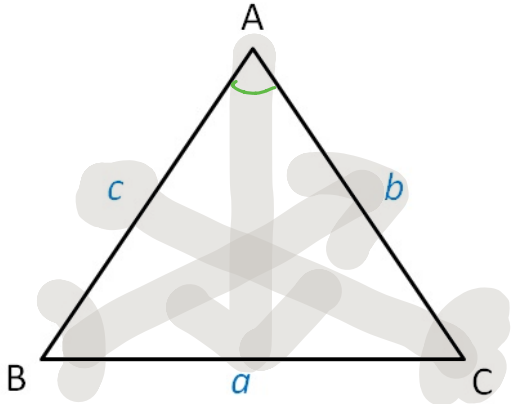
We can only use SOH CAH TOA when we are dealing with right angle trigonometry.

Many triangles we will come across will not be as favourable…

For this we have the sine law.

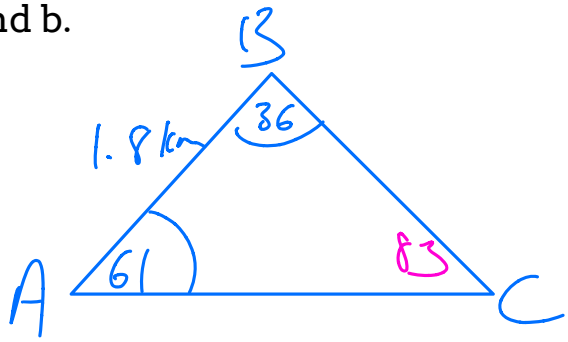
In essence: The ratio of the sides to the corresponding angle is the same for all 3 angle/side pairs in any triangle.

Sine Law



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

EG: In triangle ABC, $\angle A = 61^\circ$, $\angle B = 36^\circ$, and side c = 1.8 km. Find side a and b.



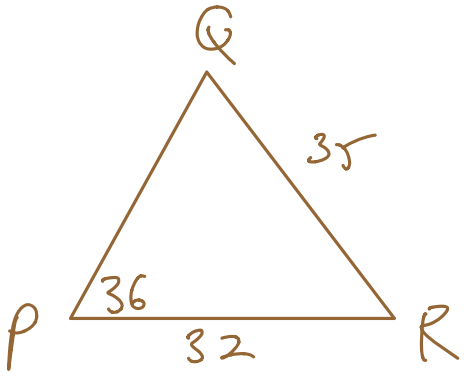
$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 83}{1.8} = \frac{\sin 61}{a} \qquad \frac{\sin 83}{1.8} = \frac{\sin 36}{b}$$

$$a = \frac{(\sin 61)1.8}{\sin 83} \qquad b = \frac{1.8(\sin 36)}{\sin 83}$$

$$a = 1.6 \text{ km} \qquad b = 1.06 \text{ km}$$

EG: In triangle PQR, $\angle P = 36^\circ$, p = 35 cm, and q = 32 cm. Determine $\angle R$ and r.



$$\frac{\sin 36}{35} = \frac{\sin Q}{32}$$

$$\frac{32(\sin 36)}{35} = \sin Q$$

$$Q = \sin^{-1}\left(\frac{32 \sin 36}{35}\right)$$

$$Q = 32.5^\circ$$

$$\frac{\sin 36}{35} = \frac{\sin R}{r}$$

$$\frac{\sin 36}{35} = \frac{\sin(111.5)}{r}$$

$$r = \frac{35(\sin 111.5)}{\sin(36)}$$

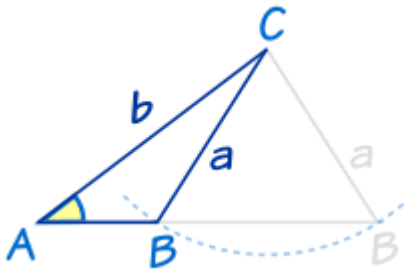
$$r = 55.4 \text{ cm}$$

The ambiguous case:

If you are given 2 angles and 1 side, then the triangle you solve for is uniquely defined. (AAS → **A**ngle **A**ngle **S**ide)

We must be aware of the ambiguous case though:

If you are given an Angle then 2 sides (ASS), we have 2 possible triangles that can be formed.



EG: In $\triangle ABC, \angle A = 32°, a = 24, b = 40$ Solve the triangle (solve means all angles and sides).

EG: In $\triangle DEF, \angle D = 29°, d = 19, e = 14$. Solve the triangle.

You try:
In $\triangle PQR, \angle P = 33°, p = 5, q = 40$. Solve.

HW: 2.3:
1ac, 2-4bc, 5