

Cross-Topic Question:

What is the period and amplitude of $y = k \cos^2 x - k \sin^2 x$

	Amp	Period
a.	k	2π
b.	k	$\frac{2\pi}{k}$
c.	k	π
d.	$\frac{k}{2}$	$\frac{2\pi}{k}$

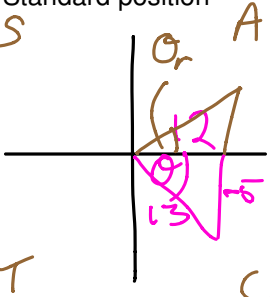
$$y = k \cos(2x)$$

↑
a

$$p = \frac{2\pi}{2} = \pi$$

Problems:

If $\sin \theta = -\frac{5}{13}$ and $\frac{3\pi}{2} < \theta < 2\pi$ find the exact value of:

<p>a. Draw the angle in Standard position</p> 	<p>Find the exact value of: $\sin 2\theta$</p> $= 2 \sin \theta \cos \theta$ $= 2 \left(-\frac{5}{13} \right) \left(\frac{12}{13} \right)$ $= -\frac{120}{169}$	<p>Find the exact value of: $\tan 2\theta$</p> $= \frac{2 \tan \theta}{1 - \tan^2 \theta}$ $= \frac{2 \left(-\frac{5}{12} \right)}{1 - \left(-\frac{5}{12} \right)^2}$ $= -\frac{120}{119}$	<p>Find the exact value of: $\cos 2\theta$</p> $= \cos^2 \theta - \sin^2 \theta$ $= \left(\frac{12}{13} \right)^2 - \left(-\frac{5}{13} \right)^2$ $= \frac{119}{169}$
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In Calculus (which is really why this whole unit exists ©) we will also write our $\cos 2\theta$ identities in terms of sine or cosine.

<p>Solve for $\cos^2 \theta$: $\cos 2\theta = 2\cos^2 \theta - 1$</p> $\cos^2 \theta = \frac{\cos(2\theta) + 1}{2}$	<p>Solve for $\sin^2 \theta$: $\cos 2\theta = 1 - 2\sin^2 \theta$</p> $\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$
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Find an expression that is equivalent to $\sin(\pi - 2x)$

$$\begin{aligned}
 &= \sin \theta \cos \phi - \cos \theta \sin \phi \\
 &= \sin \pi \cos(2x) - \cos \pi \sin(2x) \\
 &= 0 - (-1) \sin(2x) \\
 &= \sin(2x)
 \end{aligned}$$

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Verify: $2 \sin^3 x - \sin x = -\cos(2x)$

LEFT SIDE

RIGHT SIDE

$$\begin{aligned}
 &= \sin(2 \sin^2 - 1) \\
 &= -\sin(1 - 2 \sin^2) \\
 &= -\sin(\cos(2x))
 \end{aligned}$$

$$-\cos(2x)$$

When is this true?

$$-\sin \cos(2x) + \cos(2x) = 0$$

$$\cos(2x)(1 - \sin x) = 0$$

$$= 0$$

$$\begin{aligned}
 &= 0 \\
 &1 - \sin(x) > 0 \\
 &\sin(x) = 1 \\
 &x = \frac{\pi}{2}
 \end{aligned}$$

$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4} + \frac{n\pi}{2}$$

check @ $\frac{\pi}{2}$

LHS	RHS
$= 2 - 1$	$= -(-1)$
$= 1$	$= 1$

$$\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$$

$$\text{Prove: } \frac{\tan x}{\sec x + 1} = \frac{2 \cos x - 2 \cos^2 x}{\sin 2x}$$

LEFT SIDE	RIGHT SIDE
	$= \frac{\cancel{2} \cos (1 - \cos)}{\cancel{2} \sin x \cancel{\cos x}}$ $= \frac{1 - \cos}{\sin}$ $= \frac{1 - \frac{1}{\sec}}{\sin}$ $= \frac{\sec - 1}{\sec \sin} = \frac{(\sec - 1) \cos}{\sin}$ $= \frac{\sec - 1}{\tan} \left(\frac{\sec + 1}{\sec + 1} \right)$ $= \frac{\sec^2 - 1}{\tan(\sec + 1)}$ $= \frac{\cancel{\tan}^2}{\cancel{\tan}(\sec + 1)}$ $= \frac{\tan}{\sec + 1}$