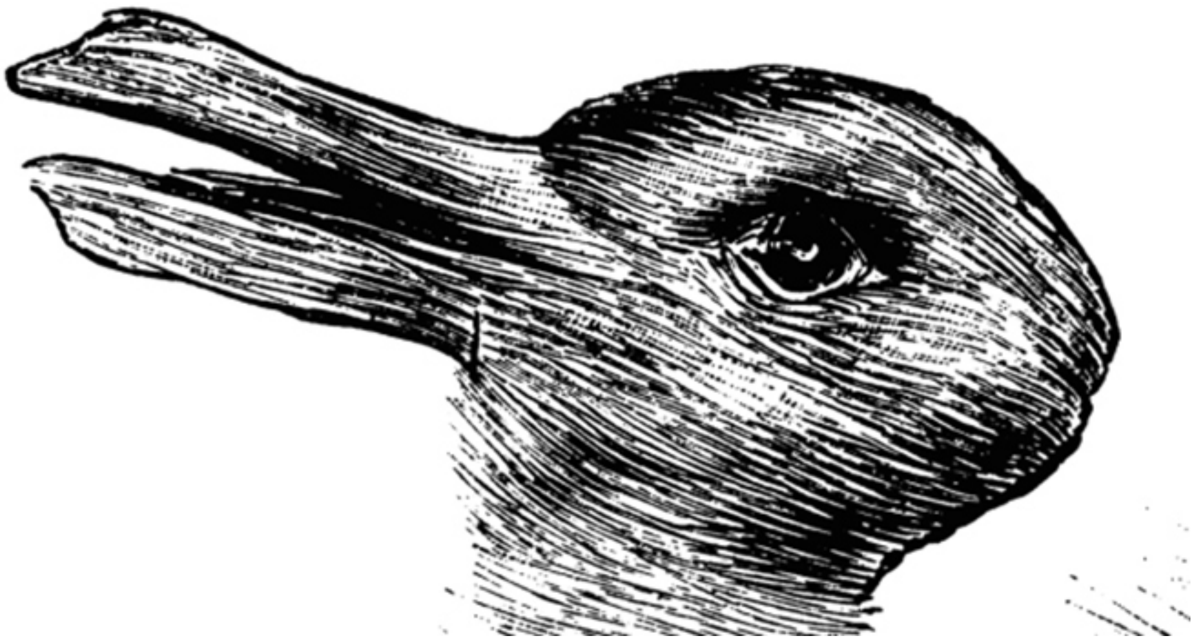
Ambiguous Case



If we are given 2 angles and 1 side (AAS) - the triangle is uniquely defined.

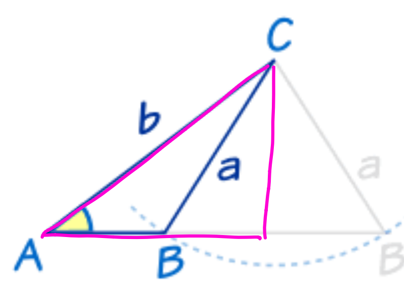
If we are given 1 angle and 2 sides (ASS) - We have (potentially) more than one possible triangle construction. This is called the ambiguous case.

In $\triangle ABC, \angle A$ and side b are constant. We have 4 scenarios depending on the length of a.

$$sin(A) = \frac{\text{opp}}{\text{hyp}}$$

$$sin(A) = \frac{h}{b}$$

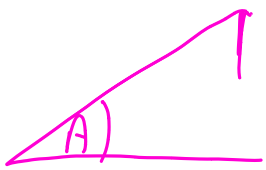
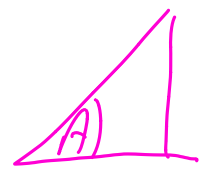
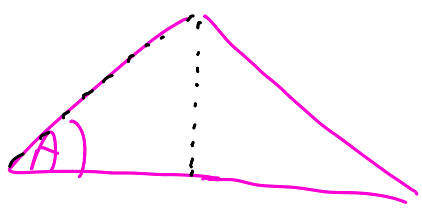
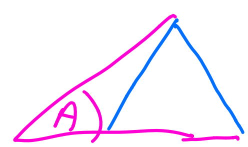
Note: $h = sin(A)b$



$h = \sin(A) b$

If $\angle A < 90^\circ$:

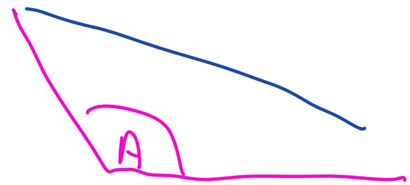
fn

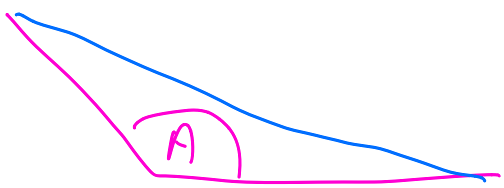
Case 1	Case 2
a < h a < sin(A)b  no soln No solutions	a = h a = sin(A)b  One Solution
Case 3	**Case 4**
$a \geq b$  One Solution	h < a < b sin(A)b < a < b  Two Solutions

Either memorize these 4 cases or work it out each time you come across these types of questions.

If $\angle A > 90^\circ$:

<table>
<tr><th>Case 1</th><th>Case 2</th></tr>
<tr><td>a < b


No Solution</td><td>a = b


No Solution</td></tr>
<tr><th colspan="2">Case 3</th></tr>
<tr><td colspan="2">a > b


One Solution</td></tr>
</table>

EG:

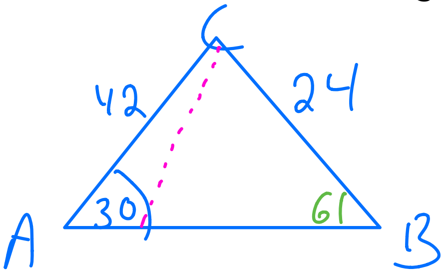
In $\triangle ABC \angle A = 30°$ a = 24 cm and b = 42 cm. Solve the triangle.

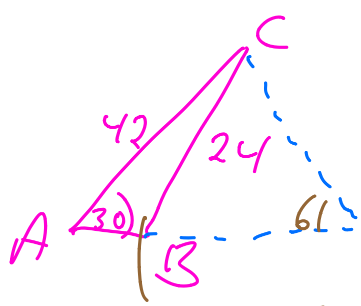
Steps:

1. Define the angle:
    a. Acute, Right, Obtuse
2. Determine the number of solutions
    a. Use logic, ($h = sin(A)b$)or refer to the chart above
        i. 0
        ii. 1
        iii. 2
3. Solve the Triangle

$$h = \sin A\, b$$
$$= \sin(30)\, 42$$
$$= 21$$

$\therefore$ 2 soln





$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 30}{24} = \frac{\sin B}{42}$$
$$B = \sin^{-1}\left(\frac{42 \cdot \frac{1}{2}}{24}\right)$$
$$B = 61°$$

$$\angle C = 180 - 61 - 30$$
$$= 89°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 30}{24} = \frac{\sin 89}{c}$$
$$c = \frac{\sin 89 (24)}{\sin 30} \rightarrow 48$$

$$\angle B = 180 - 61$$
$$= 119°$$

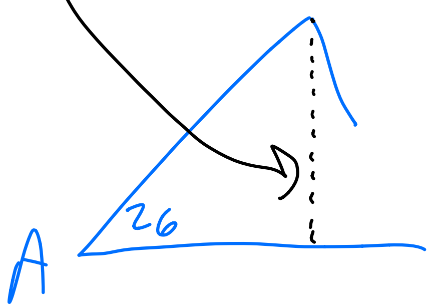
$$\angle C = 180 - 30 - 119$$
$$= 31°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\frac{1}{2}}{24} = \frac{\sin 31}{c}$$
$$c = \sin 31 (48)$$
$$= 24.7 \text{ cm}$$

Solve the triangle: $\triangle ABC : \angle A = 26^\circ, a = 15, b = 49$.

$$h = \sin 26 (49)$$
$$= 21.5$$

$\therefore$ no sol^n

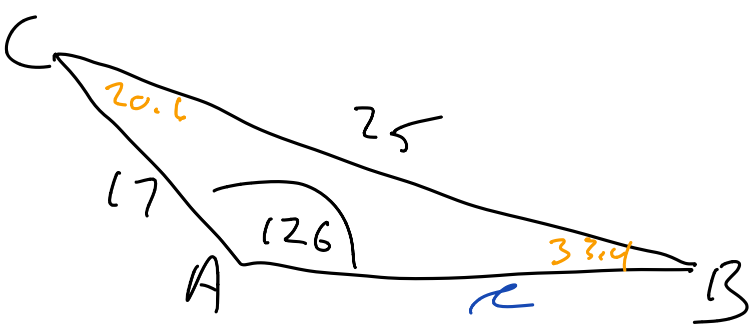


Solve the triangle: $\triangle ABC : \angle A = 126^\circ, a = 15, b = 15$.

$a = b$ $\quad \therefore$ no sol^n

Solve the triangle: $\triangle ABC : \angle A = 126^\circ, a = 25, b = 17$.

$\therefore 1$ sol^n



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 126}{25} = \frac{\sin B}{17}$$

$$B = \sin^{-1}\left(\frac{17 \sin 126}{25}\right)$$
$$= 33.4^\circ$$

$$\angle C = 180 - 126 - 33.4$$
$$= 20.6$$

In $\triangle PQR, q = 6, p = 5, \angle P = 35^{\circ}$. Determine the measure of $\angle Q$ to the nearest tenth of a degree.

HW:
6, 8, 10-13