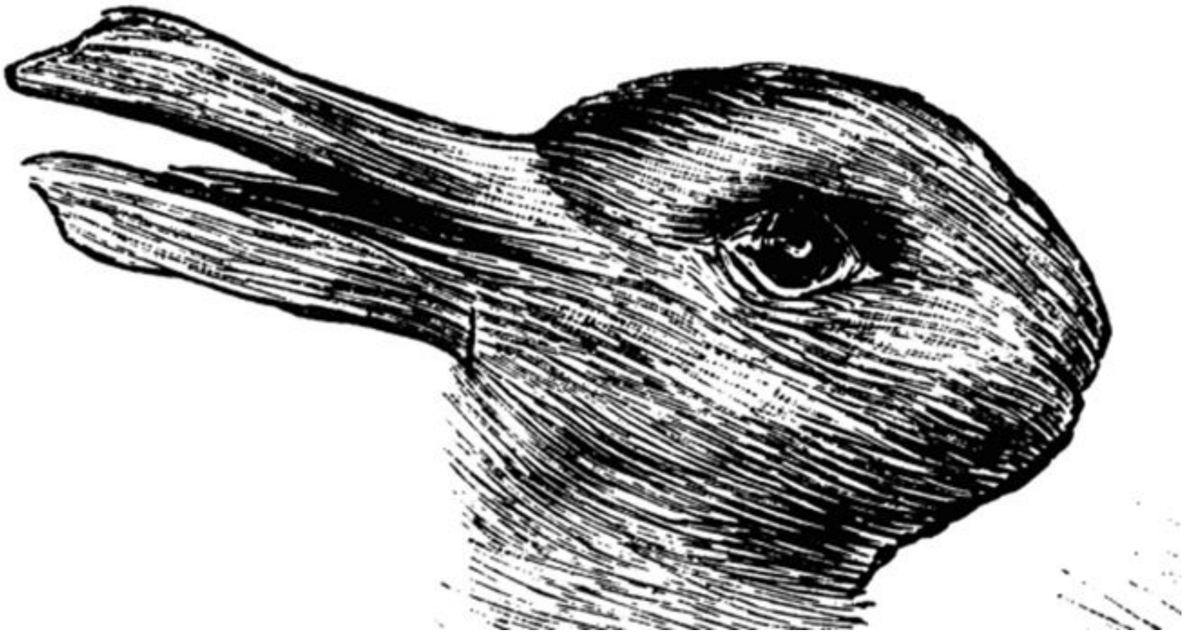
Ambiguous Case



If we are given 2 angles and 1 side (AAS) - the triangle is uniquely defined.

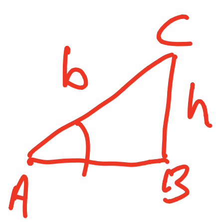
If we are given 1 angle and 2 sides (ASS) - We have (potentially) more than one possible triangle construction. This is called the ambiguous case.

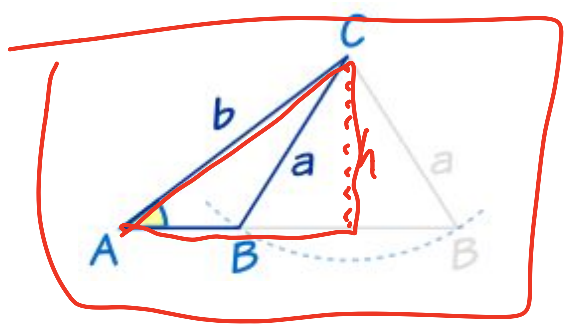
In $\triangle ABC, \angle A$ and side b are constant. We have 4 scenarios depending on the length of a.

$$sin(A) = \frac{\text{opp}}{\text{hyp}}$$

$$sin(A) = \frac{h}{b}$$

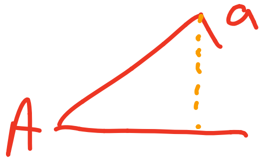
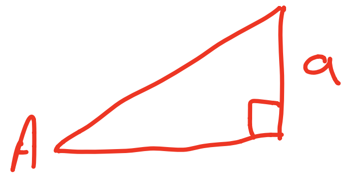
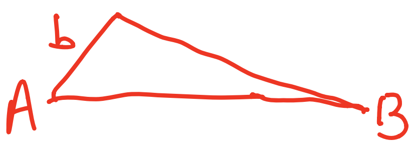
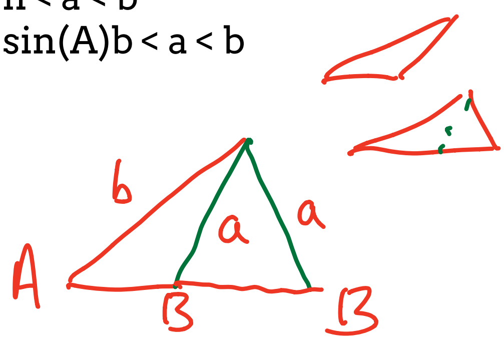
Note: $h = sin(A)b$





We need the height of our Δ.

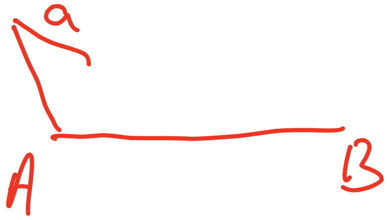
If $\angle A < 90°$:

<table>
<tr><th>Case 1</th><th>Case 2</th></tr>
<tr><td>a < h
a < sin(A)b

No solutions</td><td>a = h
a = sin(A)b

One Solution</td></tr>
<tr><th>Case 3</th><th>Case 4</th></tr>
<tr><td>$a \geq b$

One Solution</td><td>h < a < b
sin(A)b < a < b

Two Solutions</td></tr>
</table>

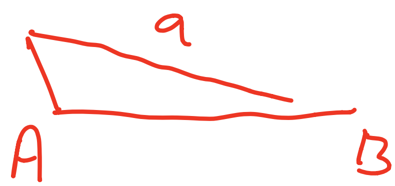
Either memorize these 4 cases or work it out each time you come across these types of questions.

If $\angle A > 90^{\circ}$:

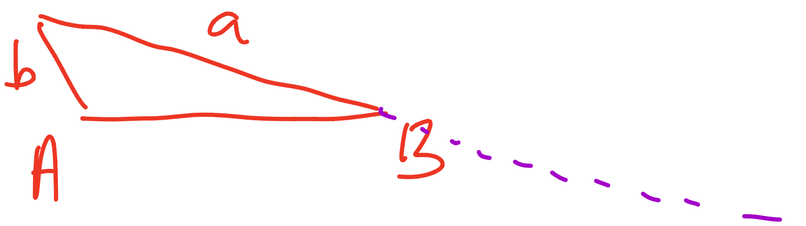
<table>
<tr><th>Case 1</th><th>Case 2</th></tr>
<tr><td>a < b



No Solution</td><td>a = b



No Solution</td></tr>
<tr><th colspan="2">Case 3</th></tr>
<tr><td colspan="2">a > b



One Solution</td></tr>
</table>

EG:

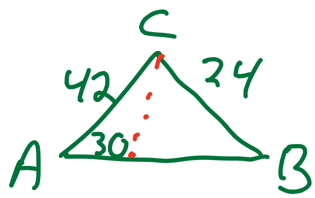
In $\triangle ABC \angle A = 30°$ a = 24 cm and b = 42 cm. Solve the triangle.

Steps:

1. Define the angle:
    a. Acute, Right, Obtuse
2. Determine the number of solutions
    a. Use logic, ($h = sin(A)b$)or refer to the chart above
        i. 0
        ii. 1
        iii. 2
3. Solve the Triangle

$$h = b\sin A$$
$$= 42 \sin(30)$$
$$= \frac{42}{2} = 21$$

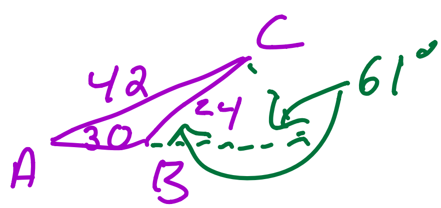
$$h < a < b$$



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 30}{24} = \frac{\sin B}{42}$$
$$B = \sin^{-1}\left[\frac{42 \cdot \frac{1}{2}}{24}\right]$$
$$\approx 61°$$

$$\angle C = 180 - 61 - 30$$
$$\approx 89°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\frac{1}{2}}{24} = \frac{\sin 89}{c}$$
$$c = 48(1)$$
$$\approx 48u$$



$$\angle B = 119°$$
$$\angle C = 180 - 30 - 119$$
$$= 31°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\frac{1}{2}}{24} = \frac{\sin 31}{c}$$
$$c = 48\sin(31)$$
$$= 25\text{cm.}$$

Solve the triangle: $\triangle ABC : \angle A = 26^\circ, a = 15, b = 49$.

$h = b \sin A$

$= 49 \sin(26)$

$\rightarrow \approx 21.5u$

$a < h$

$\therefore$ no sol^n

f^n

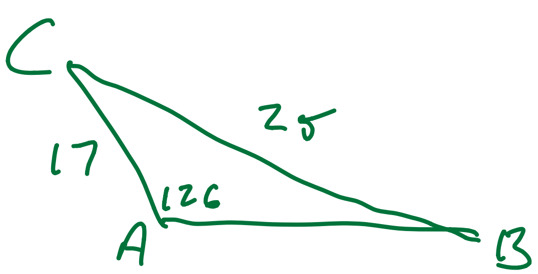
Solve the triangle: $\triangle ABC : \angle A = 126^\circ, a = 15, b = 15$.

$a = b$

$\therefore$ no sol^n

Solve the triangle: $\triangle ABC : \angle A = 126^\circ, a = 25, b = 17$.

$a > b \therefore 1 \; sol^n$



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 126}{25} = \frac{\sin B}{17}$$

$$B = \sin^{-1}\left[\frac{17 \sin(126)}{25}\right]$$

$$\approx 33.4^\circ$$

$$\angle C \approx 180 - 126 - 33.4$$

$$\approx 20.6^\circ$$

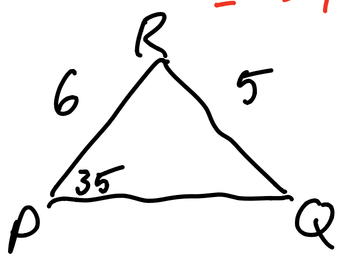
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$c \approx 10.9u$

In $\triangle PQR, q = 6, p = 5, \angle P = 35^\circ$. Determine the measure of $\angle Q$ to the nearest tenth of a degree.

$h = q\sin(P)$
$= 6\sin(35)$
$\cong 3.4$

$h < p < q$
$\therefore$ 2 sol'n



$\dfrac{\sin P}{p} = \dfrac{\sin Q}{q}$

$\dfrac{\sin 35}{5} = \dfrac{\sin Q}{6}$

$Q = \sin^{-1}\left[\dfrac{6\sin(35)}{5}\right]$
$\cong 43.5°$

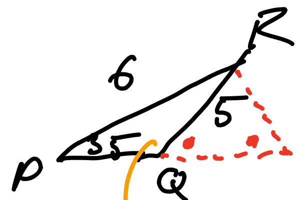
$\angle R = 180 - 43.5 - 35$
$\cong 101.5$

$\dfrac{\sin P}{p} = \dfrac{\sin R}{r}$

$\dfrac{\sin(35)}{5} = \dfrac{\sin(101.5)}{r}$

$r = \dfrac{5\sin(101.5)}{\sin(35)}$

$r \cong 8.54u.$



$180 - 43.5$
$\angle Q = 136.5$

$\angle R = 180 - 136.5 - 35$
$= 8.5$

$\dfrac{\sin P}{p} = \dfrac{\sin R}{r}$

$\dfrac{\sin 35}{5} = \dfrac{\sin(8.5)}{r}$

$r = \dfrac{5\sin(8.5)}{\sin(35)}$

$= 1.49$

HW: 6, 8, 10-13