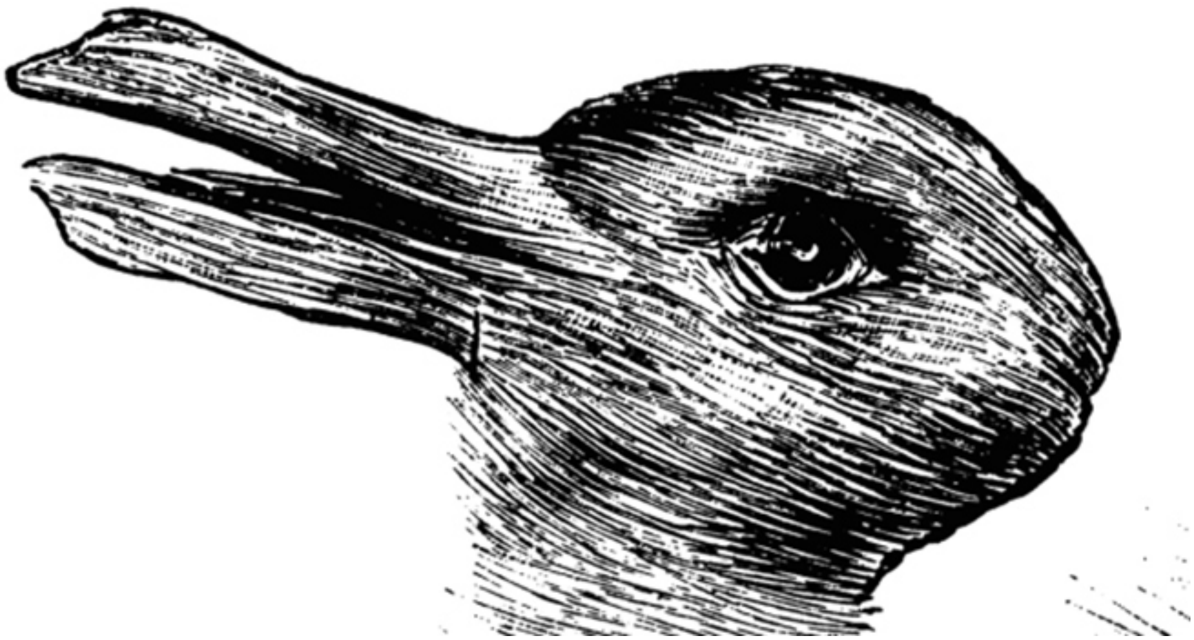
Ambiguous Case



If we are given 2 angles and 1 side (AAS) - the triangle is uniquely defined.

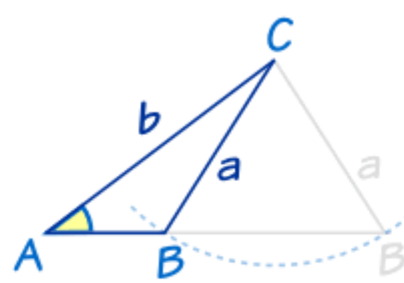
If we are given 1 angle and 2 sides (ASS) - We have (potentially) more than one possible triangle construction. This is called the ambiguous case.

In $\triangle ABC, \angle A$ and side b are constant. We have 4 scenarios depending on the length of a.

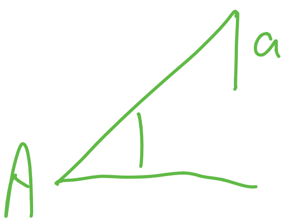
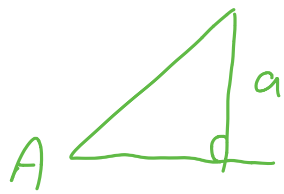
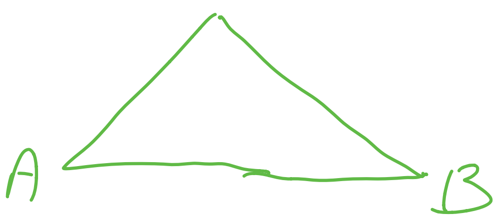
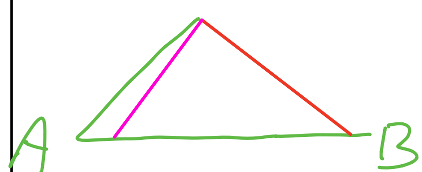
$$sin(A) = \frac{\text{opp}}{\text{hyp}}$$

$$sin(A) = \frac{h}{b}$$

Note: $h = sin(A)b$

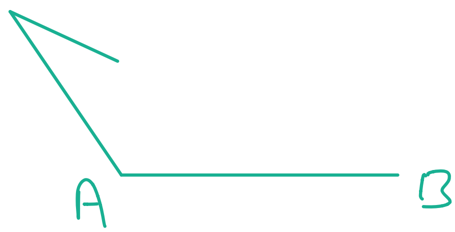
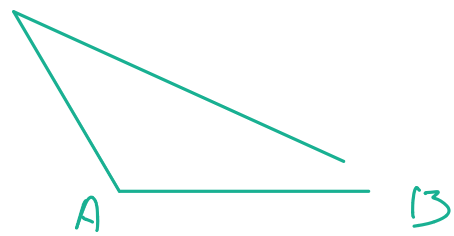
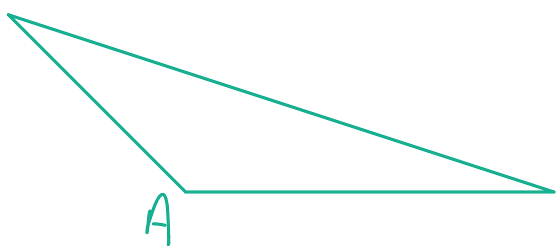


If $\angle A < 90^\circ$:

Case 1	Case 2
a < h a < sin(A)b $\therefore$ 0 sol^n  No solutions	a = h a = sin(A)b  One Solution
Case 3	**Case 4**
$a \geq b$  One Solution	h < a < b sin(A)b < a < b  Two Solutions

Either memorize these 4 cases or work it out each time you come across these types of questions.

If $\angle A > 90^\circ$:

<table>
<tr><th>Case 1</th><th>Case 2</th></tr>
<tr><td>a < b

No Solution</td><td>a = b

No Solution</td></tr>
<tr><th colspan="2">Case 3</th></tr>
<tr><td colspan="2">a > b

One Solution</td></tr>
</table>

EG:

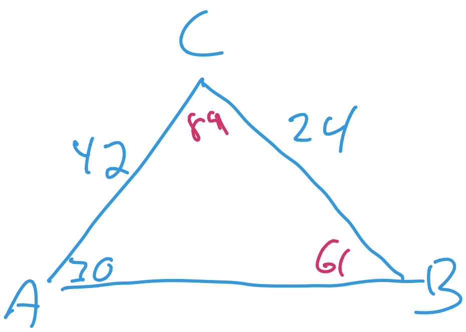
In $\triangle ABC \angle A = 30°$ a = 24 cm and b = 42 cm. Solve the triangle.

Steps:

1. Define the angle:
    a. Acute, Right, Obtuse
2. Determine the number of solutions
    a. Use logic, ($h = sin(A)b$)or refer to the chart above
        i. 0
        ii. 1
        iii. 2
3. Solve the Triangle

$\sin 30 (42) = 21 = h$

$h < a < b$



$$\frac{\sin 30}{24} = \frac{\sin B}{42}$$

$$B = \sin^{-1}\left(\frac{42 \sin 30}{24}\right)$$

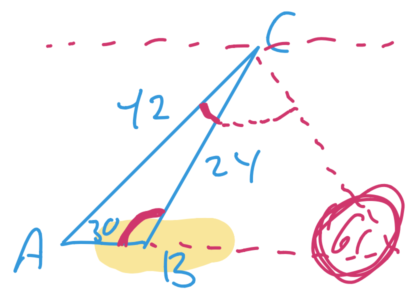
$$= 61°$$

$$C = 180 - 61 - 30$$
$$= 89°$$

$$\frac{\sin 30}{24} = \frac{\sin 89}{c}$$

$$c = \frac{24 \sin 89}{\sin 30}$$

$$c = 48 \text{ cm}$$



$$\angle B = 180 - 61$$
$$= 119°$$

$$\angle C = 180 - 119 - 30$$
$$= 31°$$

$$\frac{\sin 30}{24} = \frac{\sin 31}{c}$$

$$c = \frac{24 \sin 31}{\sin 30}$$

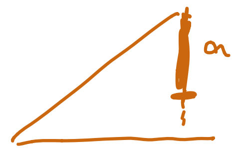
$$c = 24.7 \text{ cm}$$

Solve the triangle: $\triangle ABC : \angle A = 26°, a = 15, b = 49$.

$h = b \sin A$

$= 49 \sin(26)$

$= 21.5$



$\therefore$ no solⁿ

Solve the triangle: $\triangle ABC : \angle A = 126°, a = 15, b = 15$.

$h = b \sin A$

$= 15 \sin 126$

$= 12.1$

$a = b$

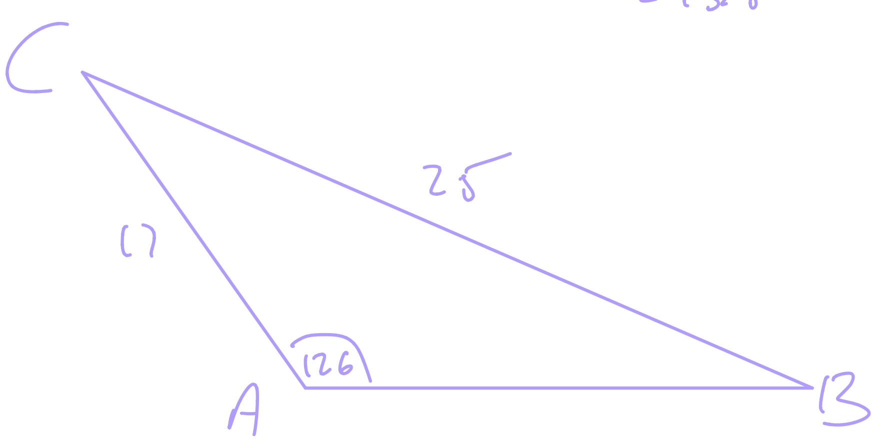
$\therefore$ no solⁿ

Solve the triangle: $\triangle ABC : \angle A = 126°, a = 25, b = 17$.

$h = b \sin A$

$= 17 \sin 126$

$= 13.8$



$\frac{\sin 126}{25} = \frac{\sin B}{17}$

$B = \sin^{-1}\left(\frac{17 \sin 126}{25}\right)$

$= 33.4°$

$C = 180 - 126 - 33.4$

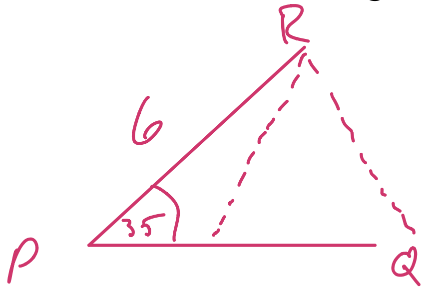
$= 20.6°$

$$\frac{\sin 126}{25} = \frac{\sin 20.6}{c}$$

$$c = \frac{25 \sin 20.6}{\sin 126}$$

$c = 10.9$ cm

In $\triangle PQR, q = 6, p = 5, \angle P = 35^\circ$. Determine the measure of $\angle Q$ to the nearest tenth of a degree.



$h = q \sin P$

$= 6 \sin 35$

$= 3.4$

HW:
6, 8, 10-13