Completing the Square

We have seen the advantages of having our quadratic equation in vertex form.

$$f(x) = a(x - p)^2 + q$$

This is very nice for graphing as we can find the shifts, stretches, and reflection very easily.

Completing the square is a useful technique for getting the vertex form of the equation when you begin with the standard form.

$$f(x) = Ax^2 + Bx + C$$

Example 1:

Complete the square for:

$$\boxed{y = x^2 - 8x + 5}$$

Here's how to do it:

1. Group the x-terms together.
2. Divide the 'x' coefficient by 2, Then square it. ie: $\left(\frac{B}{2}\right)^2$
3. Add and subtract that value to your equation.
4. The result will be a perfect square. ie: easy factoring.

$$y = (x^2 - 8x) + 5$$

$$y = (\boxed{x^2 - 8x + 16} - 16) + 5$$

$$y = (x-4)^2 - 16 + 5$$

$$y = (x-4)^2 - 11$$

$$a = 1 \quad p = 4 \quad q = -11$$

goal $\rightarrow$ Standard to vertex via Completing the Square

$$y = \left(x^2 + 6x\right) + 5$$

$$\underline{+9} - 9$$

$$y = (x+3)^2 - 9 + 5$$

$$y = (x+3)^2 - 4$$

You try:

$f(x) = x^2 - 10x + 6$ ||| $y = x^2 - 4x - 3$

$= (x-5)^2 - 25 + 6$	$y = (x-2)^2 - 4 - 3$
$= (x-5)^2 - 19$	$y = (x-2)^2 - 7$

Re-write into vertex form:

aka: complete the square to get standard form into vertex form.

$$f(x) = x^2 + 5x - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25 - 8}{4}$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{17}{4}$$

What if the coefficient in front of x^2 isn't equal to one?

Complete the square:

$$f(x) = 3x^2 - 12x - 9$$

$$3(x^2 - 4x)$$

$$3(x^2 - 4x + 4 - 4)$$

$$3[(x-2)(x-2) - 4]$$

$$= 3(x^2 - 4x) - 9$$

$$= 3(x-2)^2 - 4(3) - 9$$

$$= 3(x-2)^2 - 21$$

$$y = -x^2 + 6x - 7$$

$$y = -(x^2 - 6x) - 7$$

$$y = -(x-3)^2 - 9(-1) - 7$$

$$y = -(x-3)^2 + 2$$

$$f(x) = -2x^2 + 8x - 5$$

$$f(x) = -2(x^2 - 4x) - 5$$

$$= -2(x-2)^2 - 4(-2) - 5$$

$$= -2(x-2)^2 + 3$$

You try this one:

$$f(x) = 3x^2 + 9x - 2$$

hint: keep the fractions. Fractions are your friend.

- → complete the square
- → state the vertex
- → max or min
- → axis of symmetry
- → domain
- → range

$$f(x) = 3(x^2 + 3x) - 2$$
$$= 3\left(x + \frac{3}{2}\right)^2 - \frac{9(3)}{4} - 2$$
$$= 3\left(x + \frac{3}{2}\right)^2 - \frac{27 - 8}{4}$$
$$= 3\left(x + \frac{3}{2}\right)^2 - \frac{19}{4}$$

$\left(-\frac{3}{2}, -\frac{19}{4}\right)$

min @ $y = -\frac{19}{4}$

$x = -\frac{3}{2}$

$\{x \mid x \in \mathbb{R}\}$ $\{y \mid y \geq -\frac{19}{4}, y \in \mathbb{R}\}$

HW: pg192, Q:2,5,6,7,12ace (coefficient of one)
pg192, Q:3,4,6bcd,7bc,8bc (not a coefficient of one)