Completing the Square

We have seen the advantages of having our quadratic equation in vertex form.

$$f(x) = a(x-p)^2 + q$$

This is very nice for graphing as we can find the shifts, stretches, and reflection very easily.

Completing the square is a useful technique for getting the vertex form of the equation when you begin with the standard form.

$$f(x) = Ax^2 + Bx + C$$

Example 1:

Complete the square for:

$$y = x^2 - 8x + 5$$

Here's how to do it:

1. Group the x-terms together.
2. Divide the 'x' coefficient by 2, Then square it. ie: $\left(\frac{B}{2}\right)^2$
3. Add and subtract that value to your equation.
4. The result will be a perfect square. ie: easy factoring.

$$\left(\frac{-8}{2}\right)^2 = 16$$

$$y = [x^2 - 8x] + 5$$

$$y = [x^2 - 8x + 16] - 16 + 5$$

$$y = (x-4)(x-4) - 16 + 5$$

$$y = (x-4)^2 - 11$$

$\left(\frac{B}{2}\right)^2$

$y = x^2 + 6x + 5$

$y = \underline{x^2 + 6x + 9} - 9 + 5$ $\quad \left(\frac{6}{2}\right)^2 = 9$

$y = (x+3)(x+3) - 9 + 5$

$y = (x+3)^2 - 4$

You try:

$f(x) = x^2 - 10x + 6$ | $y = x^2 - 4x - 3$

$\left(\frac{-10}{2}\right)^2 = 25$ | $\left(\frac{-4}{2}\right)^2 = 4$

$f(x) = \underline{x^2 - 10x + 25} - 25 + 6$ | $y = \underline{x^2 - 4x + 4} - 4 - 3$

$= (x-5)^2 - 19$ | $y = (x-2)^2 - 7$

Re-write into vertex form:

aka: complete the square to get standard form into vertex form.

$$\left(\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$f(x) = x^2 + 5x - 2$

$$f(x) = (x + \tfrac{5}{2})^2$$

$$f(x) = \underbrace{x^2 + 5x + \frac{25}{4}} - \frac{25}{4} - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25 - 8}{4}$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{33}{4}$$

1,

What if the coefficient in front of x^2 isn't equal to one?

Complete the square:

$f(x) = 3x^2 - 12x - 9$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = 4$

$f(x) = 3(x^2 - 4x) - 9$

$f(x) = 3(x^2 - 4x + 4 - 4) - 9$

$f(x) = 3((x-2)^2 - 4) - 9$

$f(x) = 3(x-2)^2 - 4(3) - 9$

$f(x) = 3(x-2)^2 - 21$

$y = -x^2 + 6x - 7$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$y = -(x^2 - 6x) - 7$

$y = -(x^2 - 6x + 9 - 9) - 7$

$y = -[(x-3)^2 - 9] - 7$

$y = -(x-3)^2 + 9 - 7$

$y = -(x-3)^2 + 2$

$f(x) = -2x^2 + 8x - 5$

Hint $-2(x-2)^2 + 3$

$\left(\frac{-4}{2}\right)^2 = 4$

$y = -2(x^2 - 4x) - 5$

$y = -2(x^2 - 4x + 4 - 4) - 5$

$y = -2(x-2)^2 + 8 - 5$

$y = -2(x-2)^2 + 3$

You try this one:

$\longrightarrow f(x) = 3x^2 + 9x - 2$

hint: keep the fractions. Fractions are your friend.

- ➔ complete the square
- ➔ state the vertex
- ➔ max or min & where
- ➔ axis of symmetry
- ➔ domain
- ➔ range

Hint: $3(x^2+3x)-2$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$f(x) = 3\left(x^2+3x+\frac{9}{4}-\frac{9}{4}\right)-2$

$f(x) = 3\left(\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right)-2$

$= 3\left(x+\frac{3}{2}\right)^2 - \frac{9(3)}{4} - 2$

$= 3\left(x+\frac{3}{2}\right)^2 - \frac{27-8}{4}$

$= 3\left(x+\frac{3}{2}\right)^2 - \frac{35}{4}$

HW: pg192, Q:2,5,6,7,12ace (coefficient of one)
pg192, Q:3,4,6bcd,7bc,8bc (not a coefficient of one)