$y = a(x-p)^2 + q$ $\quad$ $y = ax^2 + bx + c$

Completing the Square

We have seen the advantages of having our quadratic equation in vertex form.

$$f(x) = a(x-p)^2 + q$$

This is very nice for graphing as we can find the shifts, stretches, and reflection very easily.

Completing the square is a useful technique for getting the vertex form of the equation when you begin with the standard form.

$$f(x) = Ax^2 + Bx + C$$

Example 1:

Complete the square for:

$$y = x^2 - 8x + 5$$

Here's how to do it:

1. Group the x-terms together.
2. Divide the 'x' coefficient by 2, Then square it. ie: $\left(\frac{B}{2}\right)^2$
3. Add and subtract that value to your equation.
4. The result will be a perfect square. ie: easy factoring.

$y = (x^2 - 8x + 16) - 16 + 5$

m → 16
a → −8

$y = (x-4)(x-4) - 11$

$y = (x-4)^2 - 11$

$$y = x^2 + 6x + 5$$

$y = (x^2 + 6x + 9) - 9 + 5$

$y = (x+3)^2 - 4$

You try:

$$f(x) = x^2 - 10x + 6 \qquad y = x^2 - 4x - 3$$

hint:

$f(x) = (x-5)^2 + 6 - 25$

$= (x-5)^2 - 19$

$y = (x^2 - 4x + 4) - 4 - 3$

$y = (x-2)^2 - 7$

Re-write into vertex form:

aka: complete the square to get standard form into vertex form.

$$f(x) = x^2 + 5x - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25 - 8}{4}$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{17}{4}$$

What if the coefficient in front of x^2 isn't equal to one?

Complete the square:

$$f(x) = 3x^2 - 12x - 9$$

$$= 3(x^2 - 4x - 3)$$

$$= 3(x^2 - 4x + 4 - 4 - 3)$$

$$= 3[(x-2)^2 - 7]$$

$$= 3(x-2)^2 - 21$$

$$y = -x^2 + 6x - 7$$

$$y = -(x^2 - 6x + 7)$$

$$y = -[(x-3)^2 + 7 - 9]$$

$$y = -[(x-3)^2 - 2]$$

$$y = -(x-3)^2 + 2$$

$$f(x) = -2x^2 + 8x - 5$$

$$f(x) = -2(x^2 - 4x) \quad - 5$$

$$= -2(x-2)^2 + 8 - 5$$

$$= -2(x-2)^2 + 3$$

You try this one:

$$f(x) = 3x^2 + 9x - 2$$

hint: keep the fractions. Fractions are your friend.

- ➔ complete the square
- ➔ state the vertex
- ➔ max or min
- ➔ axis of symmetry
- ➔ domain
- ➔ range

$f(x) = 3(x^2 + 3x \quad) - 2$

$f(x) = 3\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2(3) - 2$

$f(x) = 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{2} - 2$

$f(x) = 3\left(x + \frac{3}{2}\right)^2 - \frac{27-4}{2}$

$f(x) = 3\left(x + \frac{3}{2}\right)^2 - \frac{23}{2}$

HW: pg192, Q:2,5,6,7,12ace (coefficient of one)

pg192, Q:3,4,6bcd,7bc,8bc (not a coefficient of one)