Cosine Law

Use the Sine Law if:

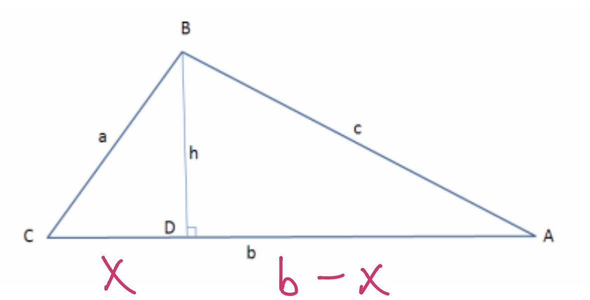
- ➔ You are given 2 angles and 1 side
- ➔ You are given 2 sides and the 1 angle is opposite a given side

Use the Cosine Law if:

- ➔ 2 Sides and the given angle is between them
- ➔ 3 sides

The cosine law is an adjustment to the pythagorean theorem.

The pythagorean theorem can only be used for right angle triangles.



$$\overline{CD} = x \text{ and } \overline{DA} = b - x$$

$$\triangle BCD : cos(C) = \frac{x}{a}$$

$$a \cdot \cos(C) = x$$

$$\text{and } a^2 = h^2 + x^2$$

$$c^2 = h^2 + (b - x)^2$$

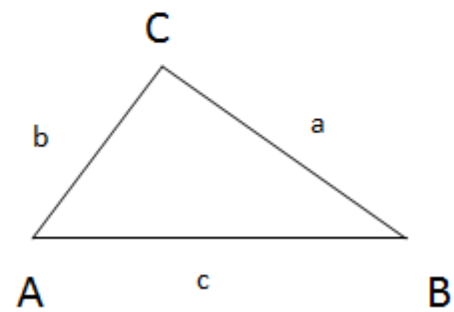
$$c^2 = h^2 + b^2 - 2bx + x^2$$

$$c^2 = h^2 + x^2 + b^2 - 2bx$$

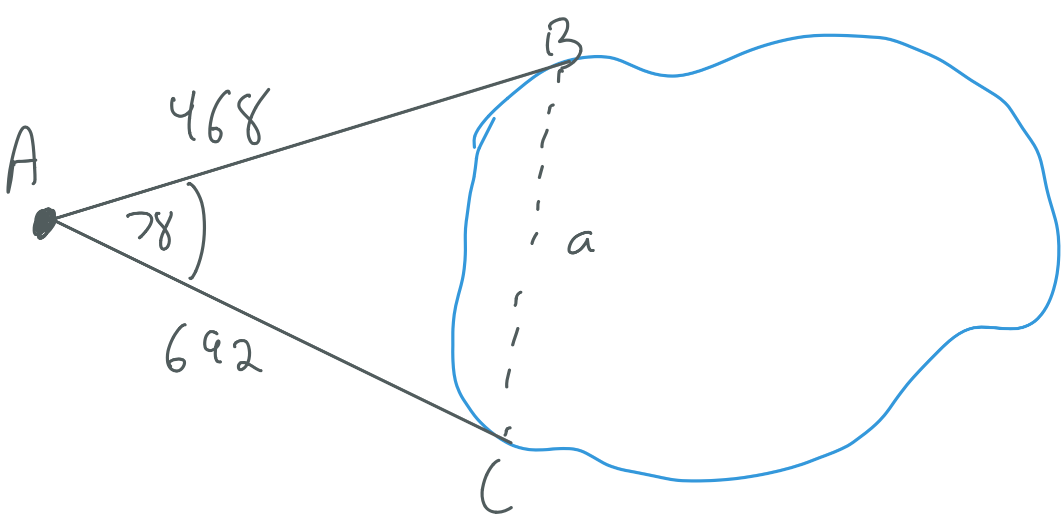
$$c^2 = a^2 + b^2 - 2ba \cdot cos(C)$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot Cos(A)$$
$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot Cos(B)$$
$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot Cos(C)$$

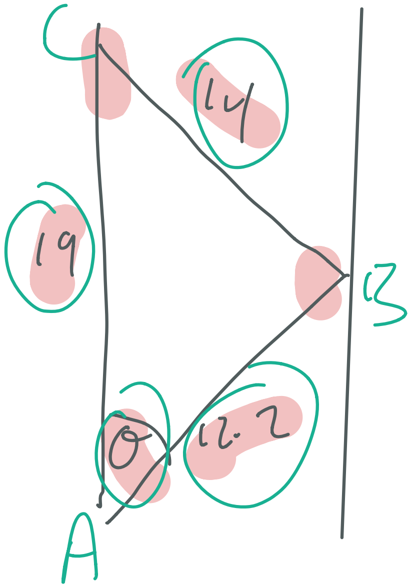


EG: A surveyor needs to find the length of a swampy area near Fish Lake. She sets her transit at point A. She measures the distance to one end of the samp as 468 m (point B) and the distance to the other side of the swamp as 692 m (point C). The angle between the two points ($\angle A$) is $78°$. Determine the length of the swampy area to the nearest meter.



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 692^2 + 468^2 - 2(692)(468)\cos 78$$
$$a = 750\text{m}$$

The Lion's Gate bridge in Vancouver is strengthened by triangular braces. Suppose the braces lengths are 14 m, 19 m, and 12.2 m. Determine the measure of the angle opposite the 14 m side to the nearest degree.

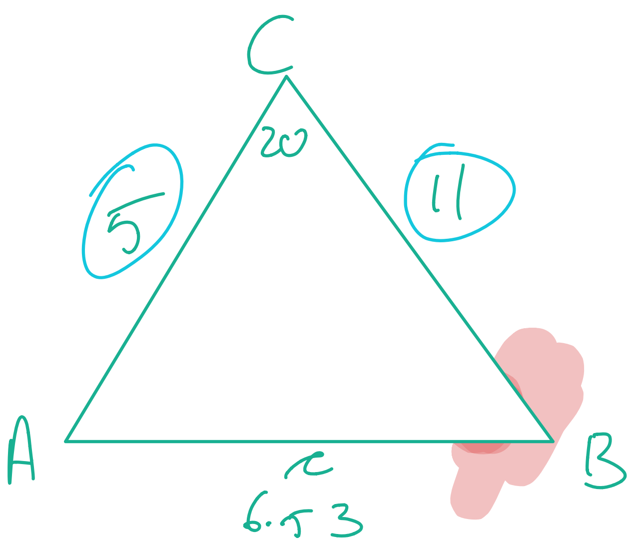


$$14^2 = 19^2 + 12.2^2 - 2(19)(12.2)\cos A$$

$$\cos^{-1}\left(\frac{14^2 - 19^2 - 12.2^2}{-2(19)(12.2)}\right) = A$$

$$47.4^\circ = A$$

In $\triangle ABC : a = 11, b = 5, \angle C = 20^\circ$. Determine the length of the unknown side and the measures of the 2 unknown angles to the nearest tenth.



$$c^2 = 5^2 + 11^2 - 2(5)(11)\cos(20)$$

$$c = 6.53$$

$$\frac{\sin A}{11} = \frac{\sin 20}{6.5295}$$

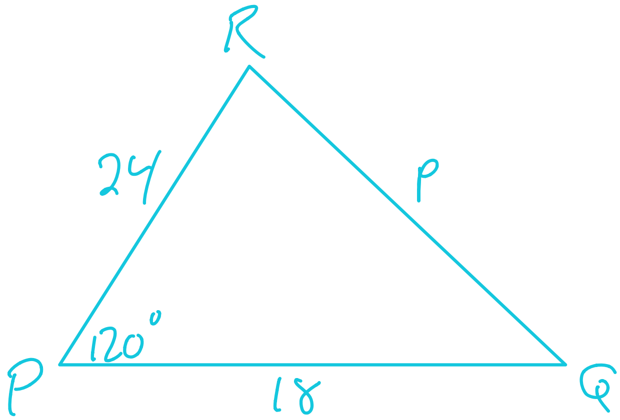
$$A = \sin^{-1}\left(\frac{11 \sin 20}{6.5295}\right)$$

$$A = 35.18^\circ$$

$$B = 180 - 35.2 - 20$$

$$= 124.8^\circ$$

In $\triangle PQR : q = 24, r = 18, \angle P = 120^\circ$. Solve the triangle.

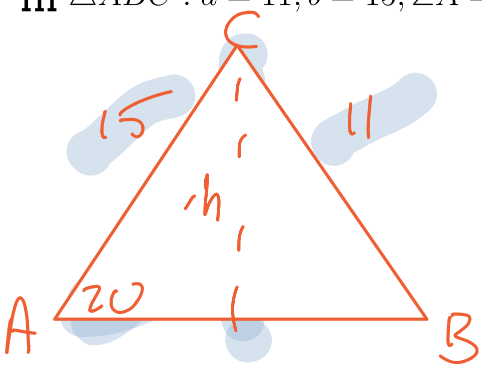


$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$p^2 = 24^2 + 18^2 - 2(24)(18)\cos(120)$$
$$p = 36.5$$

$$\frac{\sin 60}{36.5} = \frac{\sin R}{18}$$
$$R = \sin^{-1}\left(\frac{9\sqrt{3}}{36.5}\right)$$
$$R = 25.3^\circ$$

$$Q = 180 - 25.3 - 36.5$$
$$= 34.7^\circ$$

In $\triangle ABC : a = 11, b = 15, \angle A = 20^\circ$. Solve the triangle.



$$\frac{\sin B}{15} = \frac{\sin 20}{11}$$
$$B = \sin^{-1}\left(\frac{15\sin 20}{11}\right)$$
$$B = 27.8^\circ$$

$$\sin(20) = \frac{h}{15}$$
$$15\sin 20 = h$$

$h < a < b$?

HW:
1-3,7-10