The Quadratic Equation

Wouldn't it be wonderful if there was simply an equation that we could use to solve any quadratic all the time without the need to factor or complete the square or look for numbers that add to something and multiply to something else…

There is!

#SpoilerAlert

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

All those letter correspond to the quadratic equation in its standard form.

$$ax^2 + bx + c = 0$$

How did we get there? You can do it yourself…
We just complete the square on the standard form equation:

$$a(x^2 + \frac{b}{a}x) + c = 0$$

factor a.

$$\frac{\frac{b}{a}}{2} = \left(\frac{b}{2a}\right)^2$$

$$a(x + \frac{b}{2a})^2 - a[\frac{b}{2a}]^2 + c = 0$$

$$\left(\frac{b}{2}\right)^2 a$$

$$a\left(\frac{b^2}{2^2 a^{\cancel{2}}}\right)$$

$$a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c = 0$$

common denominator

$$a(x + \frac{b}{2a})^2 - \frac{b^2+4ac}{4a} = 0$$

$$a(x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$\leftarrow \sqrt{4a^2}$ $= 2a$ $\boxed{\sqrt{4a^2+1}}$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Examples: Solve using the quadratic formula:

$$3x^2+5x-2=0$$

$a=3 \quad b=5 \quad c=-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-5\pm\sqrt{25-4(3)(-2)}}{2(3)}$$

$$=\frac{-5\pm\sqrt{49}}{6}$$

$x=\frac{-5\pm7}{6}$

$x=\frac{2}{6}=\frac{1}{3} \qquad x=\frac{-12}{6}=-2$

Notice how it was the ±√ that caused us to have 2 solutions?

Under that square root is called the **discriminant**. Its value totally determines how many roots we will have.

$$b^2 - 4ac > 0 \therefore 2 \text{ solutions}$$

$$b^2 - 4ac = 0 \therefore 1 \text{ solution}$$

$$\sqrt{-1} = i$$

complex

$$b^2 - 4ac < 0 \therefore 0 \text{ real solutions}$$

Example: Determine the number of solutions:

$2x^2 - 5x + 6 = 0$	$4x^2 - 7x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$b^2 - 4ac$$

$$25 - 4(2)(6)$$

> 0

< 0

$= 0$

$\therefore$ 0 real roots

$$b^2 - 4ac$$

$$49 - 4(4)(-2)$$

$$49 + \text{blah}$$

$\therefore$ 2 answers.

Example: Solve:

$$x^2 - 2x - 1 = 0$$

You should get $x = 1 \pm \sqrt{2}$

$x = \frac{2 \pm \sqrt{8}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2}$

$= \frac{\cancel{2}(1 \pm \sqrt{2})}{\cancel{2}}$

$\sqrt{2 \cdot 4}$

$\sqrt{2} \cdot \sqrt{4}$

$\sqrt{2} \cdot 2$

$2 + 4x$

$2(1 + 2x)$

Next Class Quiz / Khan

Then test (November 8th)

HW: pg254

#2,3,4abc,7,8,21