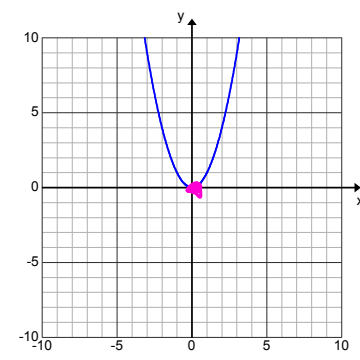
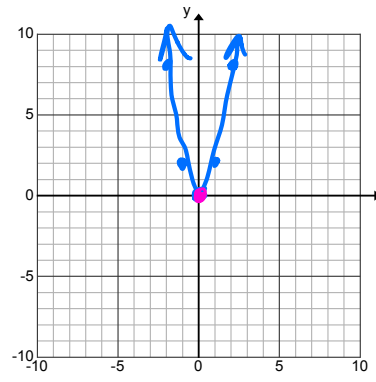
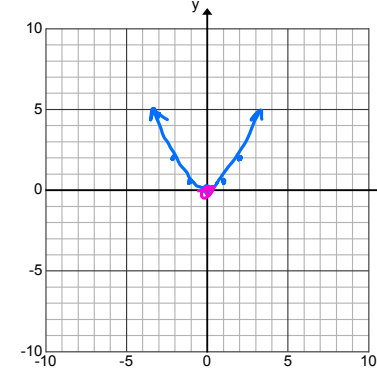


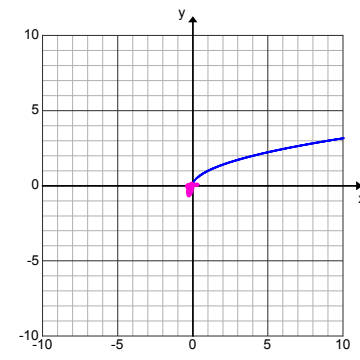
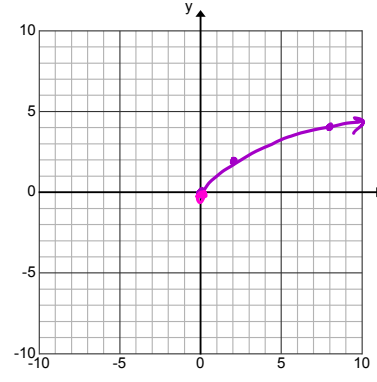
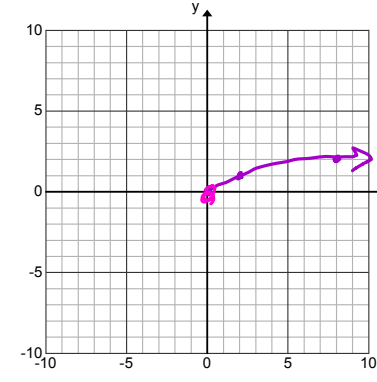
1.3 Expansions and compressions

- Graphs can be expanded/compressed vertically and horizontally

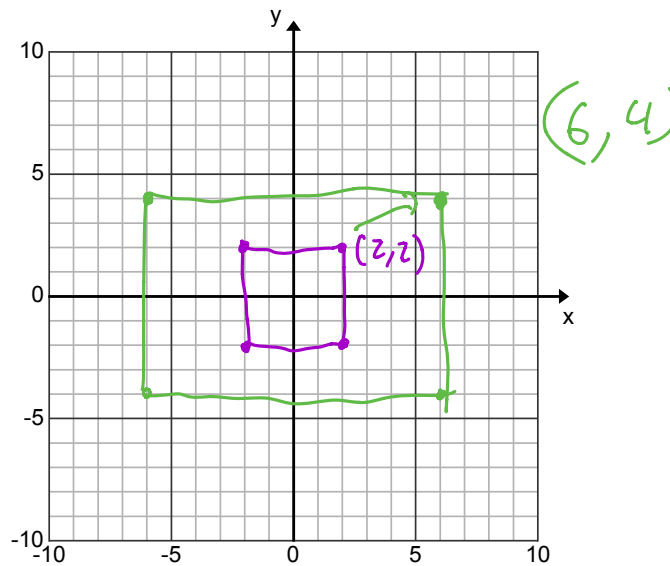
Vertical Expansions/Compressions

<p>$y = x^2$</p>  <p>Table of Values</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td><td>y</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>-1</td><td>1</td></tr> </table>	x	y	1	1	0	0	-1	1	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td>$y = 2x^2$</td> <td>$y = 2(x^2)$</td> </tr> <tr> <td>$\frac{y}{2} = x^2$</td> <td>$\frac{1}{2}y = x^2$</td> </tr> </table>  <p>Your $f(x)$ values grow <u>twice</u> as fast.</p>	$y = 2x^2$	$y = 2(x^2)$	$\frac{y}{2} = x^2$	$\frac{1}{2}y = x^2$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td>$y = \frac{1}{2}x^2$</td> <td>$y = \frac{1}{2}(x^2)$</td> </tr> <tr> <td>$2y = x^2$</td> <td>$2(y) = x^2$</td> </tr> </table>  <p>Your $f(x)$ values grow half as fast.</p>	$y = \frac{1}{2}x^2$	$y = \frac{1}{2}(x^2)$	$2y = x^2$	$2(y) = x^2$
x	y																	
1	1																	
0	0																	
-1	1																	
$y = 2x^2$	$y = 2(x^2)$																	
$\frac{y}{2} = x^2$	$\frac{1}{2}y = x^2$																	
$y = \frac{1}{2}x^2$	$y = \frac{1}{2}(x^2)$																	
$2y = x^2$	$2(y) = x^2$																	

Horizontal Expansions/Compressions

<p>$y = \sqrt{x}$</p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td><td>y</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>4</td><td>2</td></tr> </table>	x	y	0	0	1	1	4	2	<p>$y = \sqrt{2x}$</p> 	<p>$y = \sqrt{\frac{1}{2}x} = \sqrt{\frac{x}{2}}$</p> 
x	y									
0	0									
1	1									
4	2									

This also works for general functions $(f(x), g(x), k(x) \dots)$. For example, given the graph of $y = f(x)$ sketch a graph of $y = 2f(3x)$



Points on graphs can be expanded and compressed as well.

Ex. The point $(2, 3)$ is on the graph of $y = g(x)$ then what point must be on the graph of

$$y = -3g\left(\frac{1}{2}x\right)? \quad \left(\frac{2}{2}, 3(-3)\right) \rightarrow (1, -9)$$

In general:

The function $y = f(x)$ with the function $y = af(x)$ has been

- Vertically Expanded if $a > 1$ or $a < -1$
- Vertically Compressed if $-1 < a < 1$ ($a \neq 0$)

The function $y = f(x)$ with the function $y = f(bx)$ has been

- Horizontally Expanded if $-1 < b < 1$ ($b \neq 0$)
- Horizontally Compressed if $b > 1$ or $b < -1$

New Word: INVARIANT POINT is a point that doesn't move after it has been affected by a transformation.