1.5 More on Inverses

f^{-1}

Graphically there are 2 ways to find an inverse:

1. Switch x & y coordinates of the function and re-plot
2. Reflect over the line y=x $(1,2) \rightarrow (2,1)$

Algebraically there is only one way: Switch x & y in the function and solve for y.

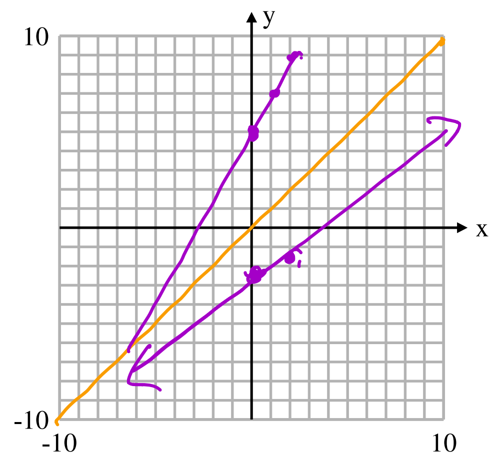
Function: $y = 2x + 5$

Find inverse:

$x = 2y + 5$

$y = \frac{x}{2} - \frac{5}{2}$

$m = \frac{1}{2}$ $b = -\frac{5}{2}$



Function: $y = \frac{2}{3}(x+1)$

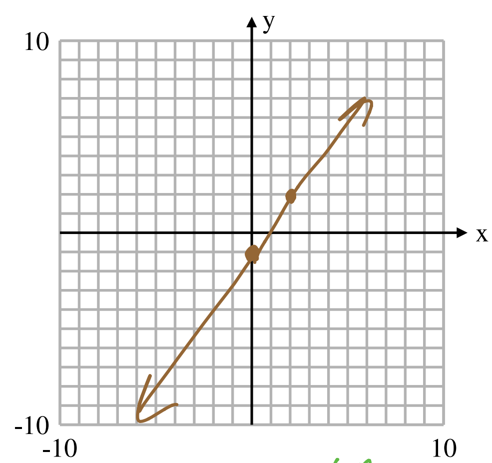
Find inverse:

$x = \frac{2(y+1)}{3}$

$m = \frac{3}{2}$

$b = -1$

$\frac{3x}{2} - 1 = y$

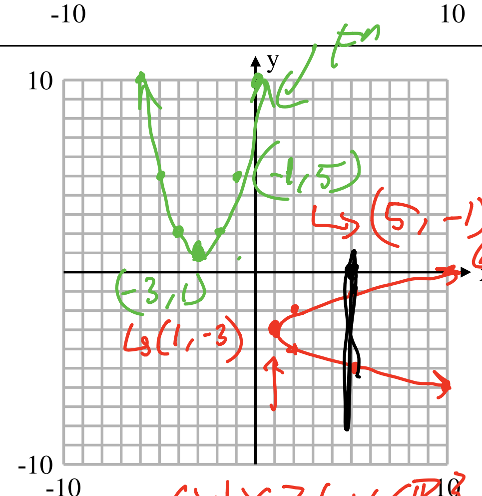


Function: $y = (x+3)^2 + 1$

Find inverse:

$x = (y+3)^2 + 1$

$\pm\sqrt{x-1} - 3 = y$



$f^{-1} \neq fn$

Domain of Function: $\{x \mid x \in \mathbb{R}\}$

Range of Function: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

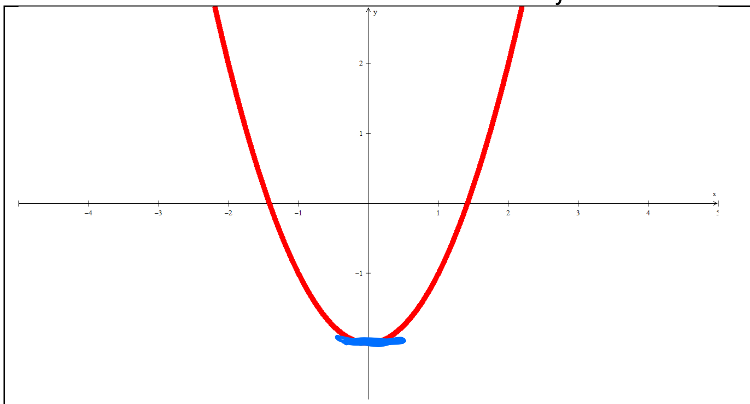
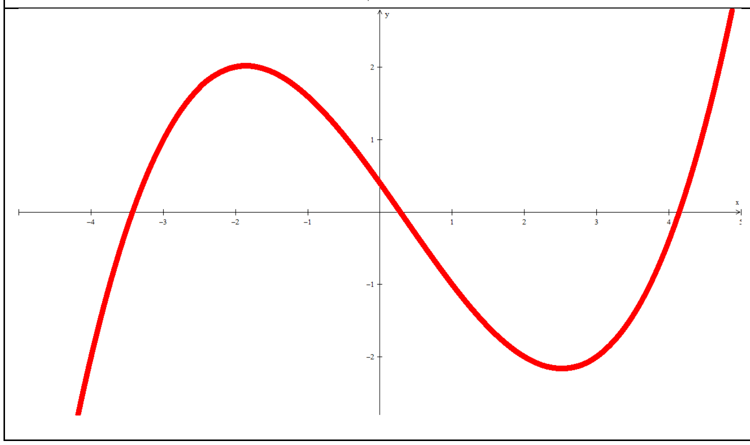
Domain of Inverse: $\{x \mid x \geq 1, x \in \mathbb{R}\}$

Range of Inverse: $\{y \mid y \in \mathbb{R}\}$

they're switched

Note: when you switch x & y in the function, most likely graph will change and so will the domain and range.

Notice that the inverse of a function isn't always a function.

	Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \geq -2, y \in \mathbb{R}\}$ Inverse Domain: $\{x \mid x \geq -2, x \in \mathbb{R}\}$ Range $\{y \mid y \in \mathbb{R}\}$
	Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \in \mathbb{R}\}$ Inverse Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \in \mathbb{R}\}$

Recall: The **graphical test** to determine if a graph represents a function is called the **vertical line test.** The test checks to see that for each x-value there is only one y-value.

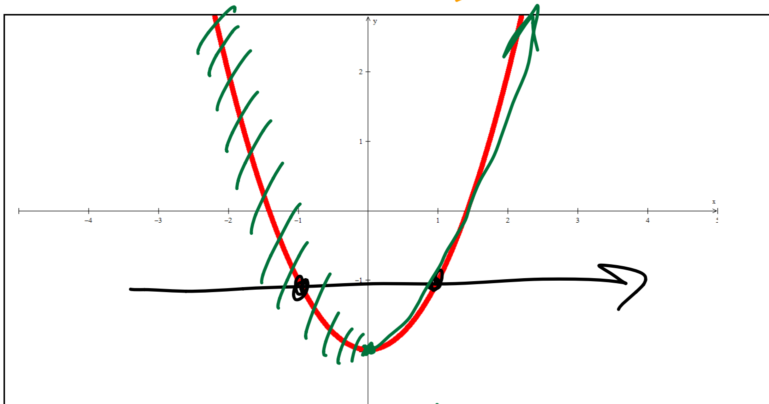
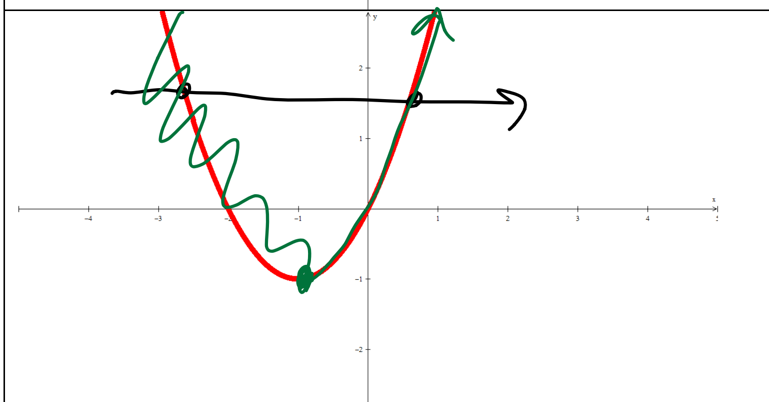
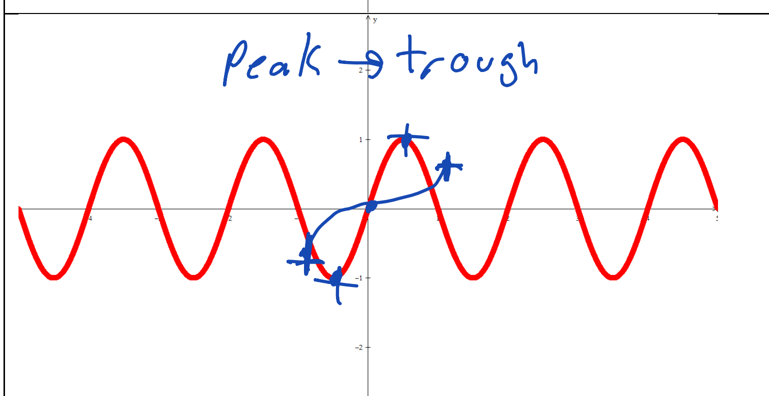
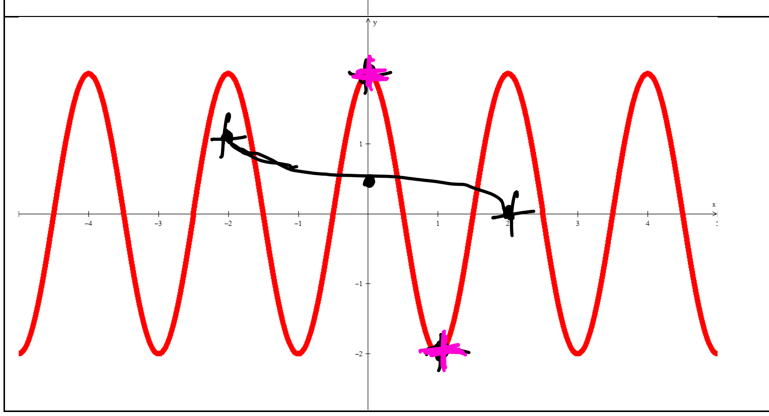
The **graphical test** to determine if the inverse of a function is a function is called the **horizontal line test**. The test checks to see that for each y-value there is only one x-value.

The **algebraic test** to determine if one function (f(x)) is the inverse of another (g(x)) is to see if: $f(g(x)) = x$ and $g(f(x)) = x$.

Think: What is going on here? Is there a relationship that is important?

If the inverse of a function isn't a function, we will restrict the domain of the original function so that it will pass the horizontal line test (and the inverse will pass the vertical line test).

Given the graph below, what can you restrict the domain to so that the inverse is a function:

	Domain: $\{x \mid x \geq 0, x \in \mathbb{R}\}$ Range: $\{y \mid y \geq -2, y \in \mathbb{R}\}$ Inverse Domain: $\{x \mid x \geq -2, x \in \mathbb{R}\}$ Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$
	Domain: $\{x \mid x \geq -1, x \in \mathbb{R}\}$ Range: $\{y \mid y \geq -1, y \in \mathbb{R}\}$ Inverse Domain: $\{x \mid x \geq -1, x \in \mathbb{R}\}$ Range: $\{y \mid y \geq -1, y \in \mathbb{R}\}$
	Domain: $\{x \mid -\frac{1}{2} \leq x \leq \frac{1}{2}, x \in \mathbb{R}\}$ Range: $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$ Inverse Domain: $\{x \mid -1 \leq x \leq 1, x \in \mathbb{R}\}$ Range: $\{y \mid -\frac{1}{2} \leq y \leq \frac{1}{2}, y \in \mathbb{R}\}$
	Domain: $\{x \mid 0 \leq x \leq 1, x \in \mathbb{R}\}$ Range: $\{y \mid -2 \leq y \leq 2, y \in \mathbb{R}\}$ Inverse Domain: $\{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$ Range: $\{y \mid 0 \leq y \leq 1, y \in \mathbb{R}\}$