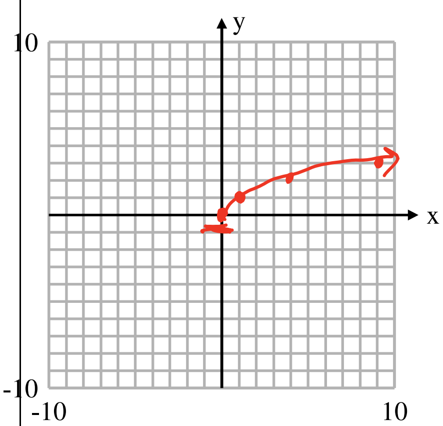
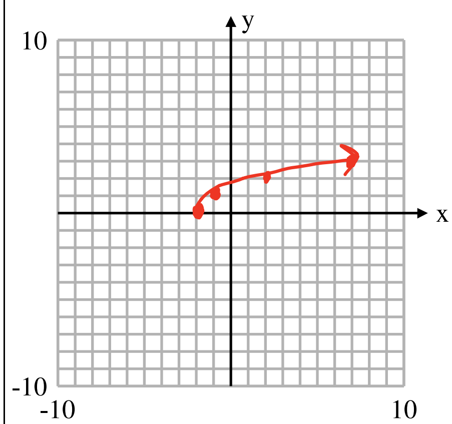
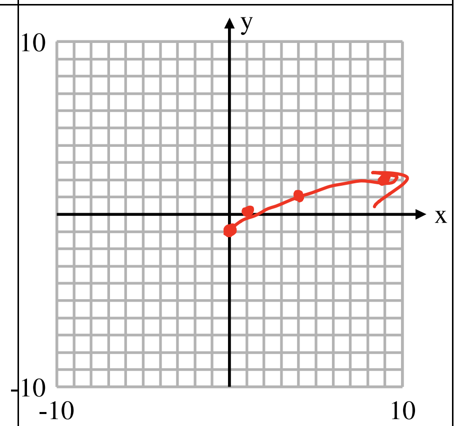
1.6 Radical Functions

A radical function is a function that involves a radical with a variable in the radicand.

Ex. $y=\sqrt{2x-5}$ $\qquad\qquad$ $y=\sqrt[3]{8x}$ $\qquad\qquad$ $y=(2x)^{\frac{1}{5}}$

Radical functions with even indices ($\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, etc) have restricted domains.

Ex. Use a table of values to graph the following functions:
Hint: remember about domain restrictions when choosing values of x for your table of values

$y=\sqrt{x}$	$y=\sqrt{x+2}$	$y=\sqrt{x}-1$
		

Graphing Radical Functions Using Transformations

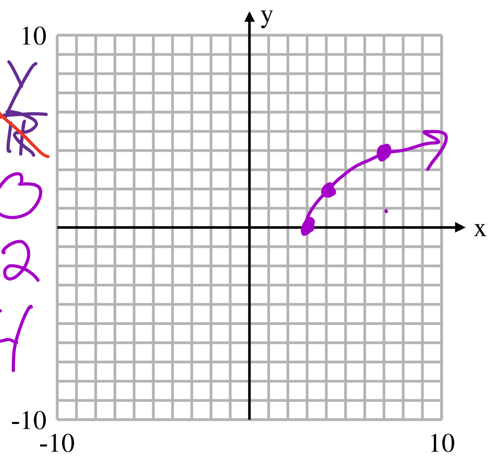
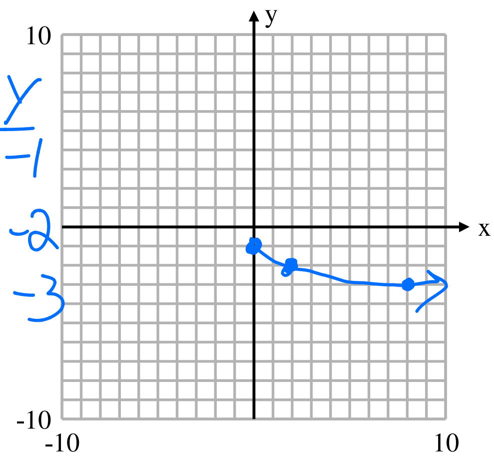
You can also graph radical functions by applying transformations to the graph of $y=\sqrt{x}$. To graph $y=a\sqrt{b(x-c)}+d$, recall the effects of the values in other types of functions.

- a - ay
- b – $\frac{x}{b}$
- c – $x-c$
- d - $y+d$

(x, y)

$\hookrightarrow \left(\frac{x}{b}-c,\ ay+d\right)$

Ex. Describe how the following graphs compare to the graph of $y=\sqrt{x}$, then sketch the graphs

$y=2\sqrt{x-3}$	$y+1=-\sqrt{\frac{1}{2}x}$
→ Vertical stretch (2) → Right 3	→ Compress x (2) → flipped over x-axis → down 1
x | y: 0 | ~~ℝ~~; 3 | 0; 4 | 2; 7 | 4 	x | y: 0 | -1; 2 | -2; 8 | -3 

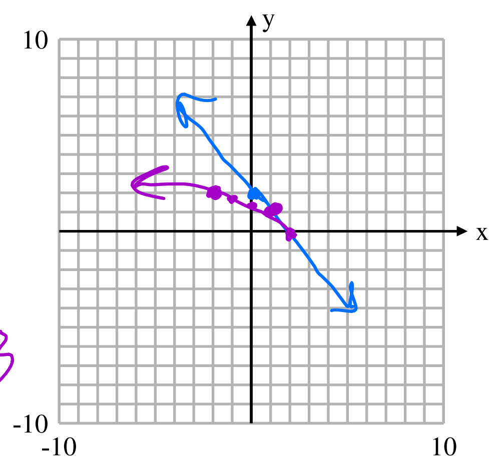
Square Root of a Function

The square root of a function, $y=f(x)$, is the function $y=\sqrt{f(x)}$.

The square root of a function is only defined for $f(x)\geq 0$.

Ex. Use a table of values to graph $y=2-x$ and $y=\sqrt{2-x}$ on the same axis.

x	$y=2-x$	$y=\sqrt{2-x}$
-2	4	2
-1	3	$\sqrt{3}$
0	2	$\sqrt{2}$
1	1	1
2	0	0



$\{x \mid x\in\mathbb{R}\}$ $\{y \mid y\in\mathbb{R}\}$

$\{x \mid x\leq 2, x\in\mathbb{R}\}$ $\{y \mid y\geq 0, y\in\mathbb{R}\}$

Identify the domain and range of each function and any invariant points.

$2-x=\sqrt{2-x}$

$4-4x+x^2=2-x$

$x=2,1$

Graphing the Square Root of a Function from the Graph of the Function

Graphing the square root of a function from the graph involves a 4 step process, as seen in the example below:

Ex. Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. $y = \sqrt{x-2}$

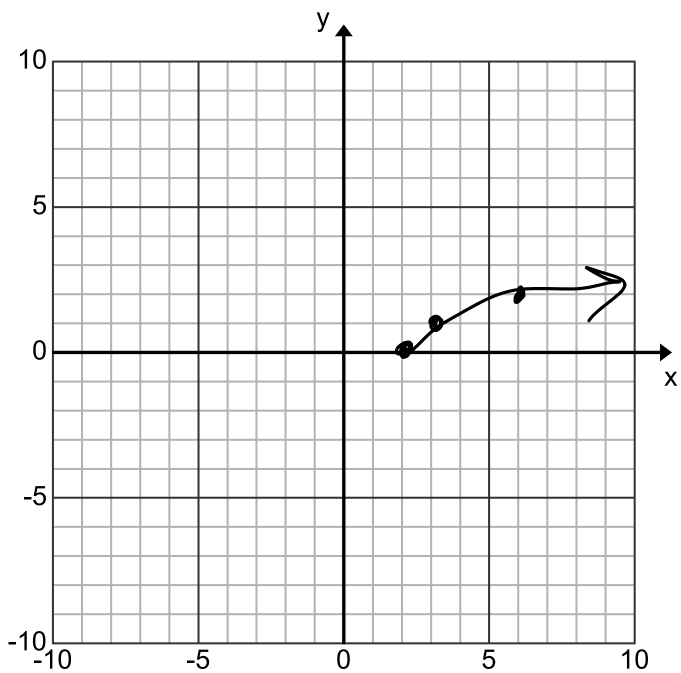
Step 1: Invariant Points Occur at y=0 and y=1 because

$y=0$ $\sqrt{f(x)}$
$= f(x)$
$y=1$ if 0,1

Step 2: Draw a curve above the graph between the invariant points (only where the graph of $y = f(x)$ is positive, of course)

Step 3: Choose a few points where the values of y are greater than 1, and square root these values to locate image points on the graph of $y = \sqrt{f(x)}$

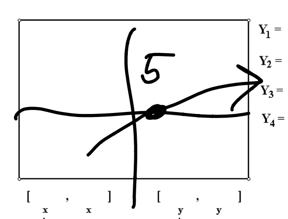
Step 4: Sketch smooth curves connecting the image points. Exclude intervals where $y = f(x)$ is negative (below the x-axis).



Solving Radical Equations Graphically (Optional)

Recall that the root(s) of an equation are equal to the *x*-intercept of the graph of the corresponding radical function.

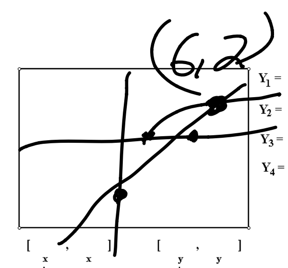
Ex. Determine the roots of $\sqrt{x+4} - 3 = 0$ graphically.



- Graph $y = \sqrt{x+4} - 3$ using technology.
- The function has an *x*-intercept at (5,0).

Sometimes it is easier to find a solution by using a system of equations and determining the intersection point.

Ex. Solve $\sqrt{x-2} = x - 4$



- Graph $y = \sqrt{x-2}$ and $y = x - 4$ using technology.
  The functions intersect at (6,2).

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