Chapter 2 Algebraic Functions & Graphing

2.1 Remainder Theorem

Recall long division: $$\begin{array}{r} 8 \\ 7\overline{)60} \\ \underline{56} \\ 4 \end{array}$$

Dividend = (divisor)(quotient) + remainder

60 = (7) * (8) + 4

If doing the long division with polynomials, there are two ways to do this: long division or synthetic division (using detached coefficients).

Ex. Divide the polynomial $x^3+7x^2-10x-15$ by $x-3$ using:

Long Division

$$\begin{array}{r} x^2 + 10x + 20 \\ x-3\overline{)x^3+7x^2-10x-15} \\ \underline{x^3-3x^2} \qquad\qquad \\ 10x^2-10x \qquad \\ \underline{10x^2-30x} \qquad \\ 20x-15 \\ \underline{20x-60} \\ 45 \end{array}$$

$f(3) = \text{remainder}$

$= (3)^3+7(3)^2-10(3)-15$

$= 45$

Synthetic Division

$$\begin{array}{r|rrrr} -3 & 1 & 7 & -10 & -15 \\ & & 3 & 30 & 60 \\ \hline & 1 & 10 & 20 & 45 \end{array}$$

$\rightarrow x^2+10x+20+\frac{45}{x-3}$

← this is quicker + easier, BUT only works in this scenario (leading coefficient and degree = 1)

If a power is missing in the dividend, it must be included using zero (0) as the coefficient

To find the remainder when a polynomial is divided by a monomial (without doing the long division), use the remainder theorem:

When a polynomial $P(x)$ is divided by $x-a$, the remainder is $P(a)$

Ex. Find the remainder when x^3-4x^2+5x-1 is divided by:

$x-2$	$x+3$
function notation: $f(2) = 2^3-4(2)^2+5(2)-1$	function notation: $f(-3) = (-3)^3-4(-3)^2+5(-3)-1$
remainder: $= 1$	remainder: -79

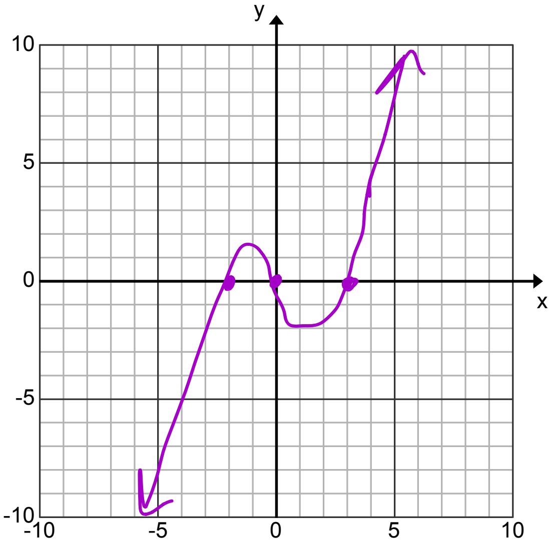
Check using synthetic division	Check using synthetic division
$\begin{array}{r\|rrrr} 2 & 1 & -4 & 5 & -1 \\ & & 2 & -4 & 2 \\ \hline & 1 & -2 & 1 & \boxed{1} \end{array}$ ✓	$\begin{array}{r\|rrrr} -3 & 1 & -4 & 5 & -1 \\ & & -3 & 24 & -78 \\ \hline & 1 & -7 & 26 & \boxed{-79} \end{array}$ ✓

If $P(a)=0$, then $x-a$ is a <u>factor</u> of the polynomial $P(x)$

Once you find factors of a polynomial, you can sketch a graph of the polynomial using the factors as roots (zeroes) of the polynomial.

Ex. Given that $P(3)$, $P(-2)$ and $P(0)$ are the only roots of a polynomial function

a. sketch a possible graph of the polynomial



b. Write a possible function

$f(x) = (x-3)(x+2)x$

x-ints @ 3, -2, 0

degree = 3

∴ cubic

Leading coefficient is >0

∴ from QIII to QI