2.2 The Factor Theorem

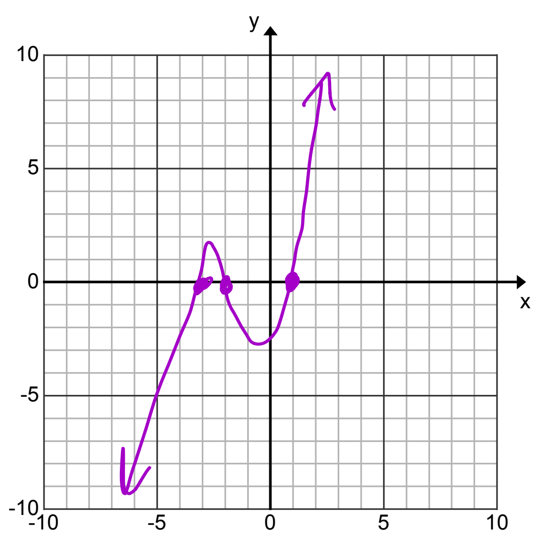
The Factor Theorem states that $(x-a)$ is a **factor** of a polynomial function $P(x)$ if, and only if, $P(a)=0$.

Example 1: Determine which binomials are factors of $P(x)=x^3+4x^2+x-6$

$(x-1)$ $P(1)=1^3+4(1)^2+1-6$ $=0$ $\therefore (x-1)$ is a factor	$(x+2)$ $P(-2)=(-2)^3+4(-2)^2+(-2)-6$ $=0$ $\therefore (x+2)$ is a factor of $P(x)$
$(x+1)$ $P(-1)=(-1)^3+4(-1)^2-1-6$ $=-4$ $\therefore x+1$ is not a factor of $P(x)$	$(x+3)$ $P(-3)=(-3)^3+4(-3)^2+(-3)-6$ $=0$ $\therefore (x+3)$ is a factor of $P(x)$

The maximum number of factors possible for $P(x)=x^3+4x^2+x-6$ is 3.
Since all the factors of the polynomial have been found, we can write the polynomial in its factored form: x^3+4x^2+x-6 = $(x-1)(x+2)(x+3)$.

Knowing that the zeros of the polynomial function $P(x)$ can give us the *x*-intercepts of the corresponding graph, we are able to sketch a graph of $P(x)=x^3+4x^2+x-6$



Example 2: **Factor** x^3+2x^2-5x-6 fully

Let $P(x)=x^3+2x^2-5x-6$

- Use the Factor Theorem to find at least one factor of $P(x)$. Try/test with values of x that are factors of 6 (the constant in the polynomial):

  ± 1 ± 2 ± 3 ± 6. ← 8 cases to check. you need to get lucky ...

$f(-1)=(-1)^3+2(-1)^2-5(-1)-6$
$=-1+2+5-6$
$=0$

$\therefore [x-(-1)]$ is a factor $(x+1)$

- To find another factor, use Synthetic (or Long) Division to divide $P(x)$ by the factor that was found in the first step. (*note* - remainder must be zero for this synthetic division too.) ← we don't want to try all 8 cases. Once you find one ... you're at a quadratic — and you know how to solve those.

$$
\begin{array}{r}
x^2+x-6 \\
x+1\,\overline{)\,x^3+2x^2-5x-6} \\
\underline{x^3+x^2} \\
x^2-5x \\
\underline{x^2-x} \\
-6x-6 \\
\underline{-6x-6} \\
0
\end{array}
$$

- If any factors are not in binomial form, factor further using synthetic division, trinomial factoring, or the quadratic formula $\left(x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$

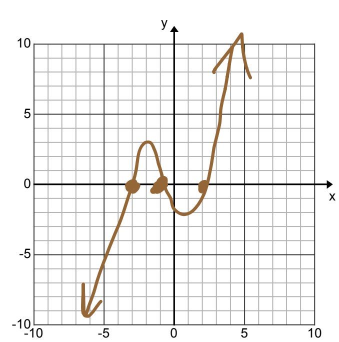
$f(x)=x^3+2x^2-5x-6$
$=(x+1)(x^2+x-6)$
↳ factor $(x-2)(x+3)$

$f(x)=(x+1)(x-2)(x+3)$

The final factored form of x^3+2x^2-5x-6 **is** $(x+1)(x-2)(x+3)$.

Sketch a graph of $y=x^3+2x^2-5x-6$ by plotting the *x* and *y* intercepts

roots @ $-1, 2, -3$
+ive Leading coefficient.



Try This:

Factor $2x^3 - 5x^2 - 4x + 3$ fully. → factors of 3 → $\pm 1, \pm 3$

4 cases

Try -1

$$f(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$$
$$= -2 - 5 + 4 + 3$$
$$= 0$$

$\therefore (x+1)$ is a factor

↳ Now long division to get a comfortable quadratic.

$$\begin{array}{r|l}
 & 2x^2 - 7x + 3 \\
\hline
x+1 & 2x^3 - 5x^2 - 4x + 3 \\
 & \underline{2x^3 + 2x^2} \\
 & \quad -7x^2 - 4x \\
 & \quad \underline{-7x^2 - 7x} \\
 & \qquad 3x + 3 \\
 & \qquad \underline{3x + 3} \\
 & \qquad\quad 0
\end{array}$$

Now $f(x) = 2x^3 - 5x^2 - 4x + 3$

$= (x+1)(2x^2 - 7x + 3)$

↳ factor this

$ab = 6$

$a + b = -7$

$-6, -1$

$= 2x^2 - 6x - x + 3$

$= 2x(x-3) - (x-3)$

$= (x-3)(2x-1)$

finally!

$$f(x) = (x+1)(x-3)(2x-1)$$

Sketch a graph of $y = 2x^3 - 5x^2 - 4x + 3$ by plotting the x and y intercepts

I have my roots

$x = -1, 3, \frac{1}{2}$

and the leading coefficients > 1.

I can make a rough graph.