2.3 Graphing Polynomials

By now, you should have a pretty good idea as to what a polynomial looks like since both sections 2.1 and 2.2 are specific to polynomials. To be more specific:

A polynomial is a mathematical expression with one or more terms, in which the exponents are whole numbers and the coefficients are real numbers.

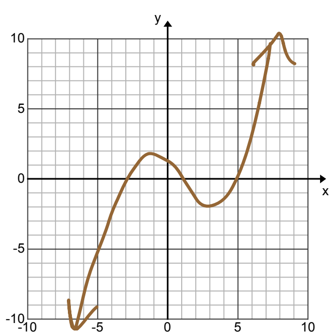
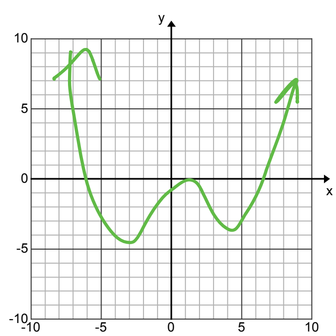
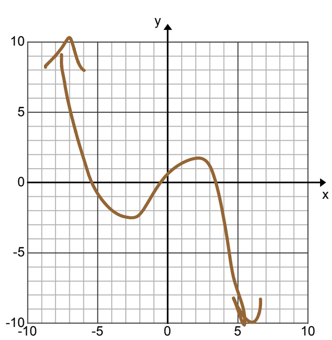
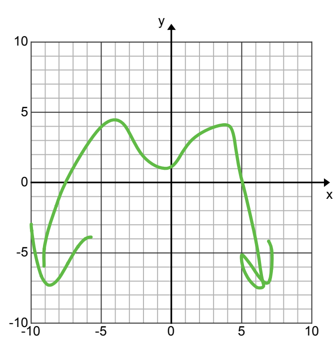
Or even more specific:

A polynomial is an expression of the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$ where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real (number) constants and n is a whole number.

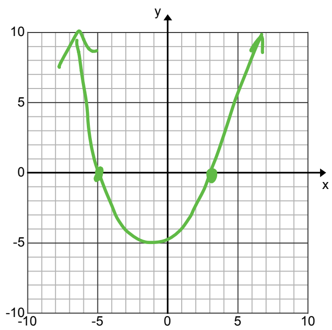
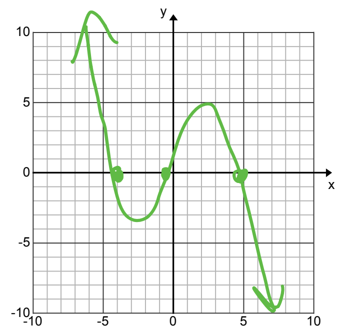
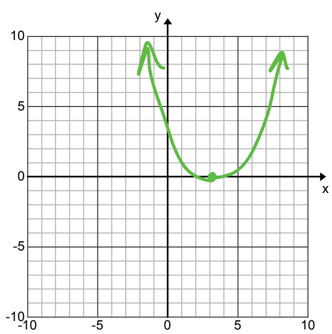
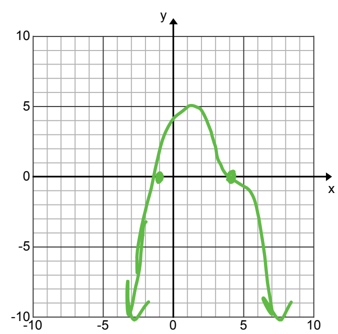
Note:

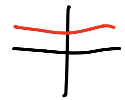
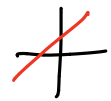
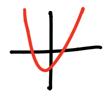
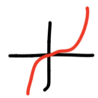
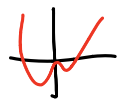
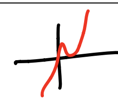
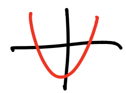
1. $f(x) = \dfrac{1}{x}$ is NOT a polynomial because it can be re-written as: $f(x) = x^{-1}$ and -1 (the exponent) is NOT a whole number.
2. $f(x) = \sqrt{x}$ is NOT a polynomial because it can be re-written as $f(x) = x^{\frac{1}{2}}$ and $\dfrac{1}{2}$ (the exponent) is NOT a whole number.

In graphing polynomial functions, it is helpful to make reference to the degree of the polynomial as the degree gives many clues to what it may look like on a graph. Knowing whether the polynomial has an even or odd degree tells us even more.

Odd function – leading coefficient positive	Even Function – leading coefficient positive
	
Odd function – leading coefficient negative	**Even function – leading coefficient negative**
	

Example 1: Sketch the following functions without using technology. Instead, use the x intercepts, the y intercept and the information you know given the degree (odd/even) to help you with your sketch. Note that the following have been factored to help you.

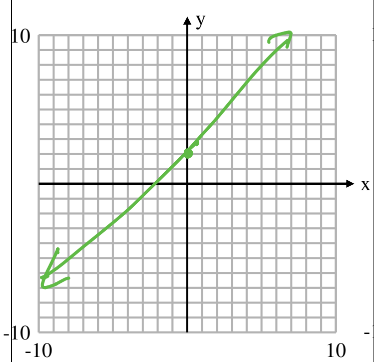
$y=(x-3)(x+5)$ 	$y=-(2x+1)(x+4)(x-5)$ 
$y=(x-3)^2$  Note: this is called a <u>double root</u>	$y=-(x+1)(x-4)^3$  Note: this is called a <u>triple root</u>

Name	Degree	Sample Equation	Max # of *x-intercepts*	Max # of Max's/Mins	Sample Graph(s)	Domain	Range
Constant	0	$y = 5$	0 (or ∞)	0		$x \in \mathbb{R}$	
Linear	1	$y = mx + b$	1	0			even degree $= y \in \mathbb{R}$
Quadratic	2	$y = ax^2 + bx + c$	2	1			odd degree look for max/min
Cubic	3	$y = ax^3 + bx^2 + cx + d$	3	2			
Quartic	4	$y = x^4 + 1$	4	3			
Quintic	5	$y = x^5 + 2$	5	4			
Sextic	6	$y = x^6 + x$	6	5			

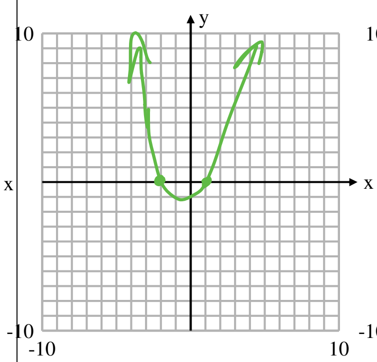
Septic	7	$y=x^7+x^6$	7	6			
Octic	8	$y=x^8-1$	8	7			
…hectic	100	$y=x^{100}$	100	99			

Graph:

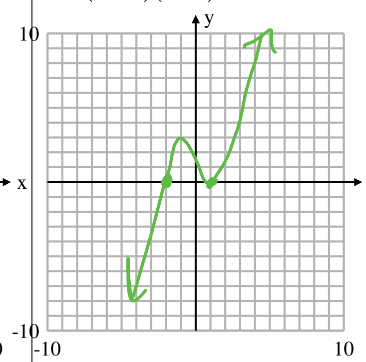
$y=(x+2)$



$y=(x+2)(x-1)$



$y=(x+2)(x-1)^2$



$y=(x+2)^2(x-1)^2(x+4)$