2.4 Operations with Functions

Mathematical operations may be performed with functions:

Sum of Functions

$h(x) = f(x) + g(x)$ is equivalent to $h(x) = (f+g)(x)$

Given $f(x) = 2x+1$ and $g(x) = x^2$ determine $h(x) = f(x) + g(x)$ $h(x) = (2x+1) + (x^2)$ $h(x) = x^2 + 2x + 1$	Given $f(x) = x^2 + 2x$ and $g(x) = x^2 + x + 2$ determine $h(x) = f(x) + g(x)$ $h(x) = (x^2+2x) + (x^2+x+2)$ $= 2x^2 + 3x + 2$

Difference of Functions

$h(x) = f(x) - g(x)$ is equivalent to $h(x) = (f-g)(x)$

Given $f(x) = 6x$ and $g(x) = x-2$ determine $h(x) = f(x) - g(x)$ $h(x) = (6x) - (x-2)$ $= 5x + 2$	Given $f(x) = -3x+7$ and $g(x) = 3x^2 - x - 2$ determine $h(x) = f(x) - g(x)$ $h(x) = (-3x+7) - (3x^2 - x - 2)$ $= -3x^2 - 2x + 9$

$f \circ g = f$ of g

Product of Functions

$h(x) = f(x)g(x)$ is equivalent to $h(x) = (f \bullet g)(x)$

Given $f(x) = 2x+5$ and $g(x) = 3x-5$ determine $h(x) = f(x)g(x)$

$$h(x) = (2x+5)(3x-5)$$
$$= 6x^2 - 10x + 15x - 25$$
$$= 6x^2 + 5x - 25$$

Given $f(x) = -2x^2 - 5x$ and $g(x) = 3x+5$ determine $h(x) = f(x)g(x)$

$$h(x) = (-2x^2 - 5x)(3x+5)$$
$$= -6x^3 - 10x^2 - 15x^2 - 25x$$
$$= -6x^3 - 25x^2 - 25x$$

Quotient of Functions

$h(x) = \dfrac{f(x)}{g(x)}$ can be written as $h(x) = \left(\dfrac{f}{g}\right)(x)$ where $g(x) \neq 0$

Given $f(x) = x^2 + x - 6$ and $g(x) = 2x+6$ determine $h(x) = \dfrac{f(x)}{g(x)}$

$$h(x) = \frac{x^2 + x - 6}{2x+6}$$
$$= \frac{\cancel{(x+3)}(x-2)}{2\cancel{(x+3)}}$$

hint: $\dfrac{x-2}{2}$

Given $f(x) = x+2$ and $g(x) = x^2 + 9x + 14$ determine $h(x) = \dfrac{f(x)}{g(x)}$

$$h(x) = \frac{x+2}{x^2 + 9x + 14}$$
$$= \frac{\cancel{x+2}}{(x+7)\cancel{(x+2)}}$$
$$= \frac{1}{x+7}$$

Domain and Range

When given the graphs of two functions we can easily graph the combined function by performing the operations on the *y*-coordinates at each point

Sketch $h(x) = f(x) + g(x)$

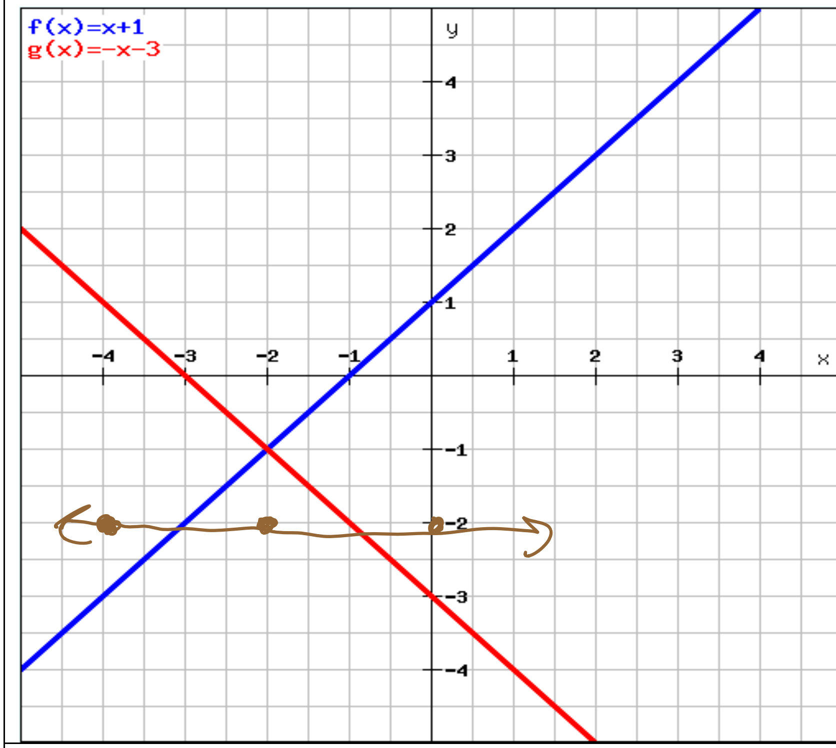


Table of Values:

x	$f(x)+g(x)$
-4	$-3+1=-2$
-2	$-1+-1=-2$
0	$1-3=-2$

Domain:

$\{x \mid x \in \mathbb{R}\}$

Range:

$\{y \mid y = -2\}$

Sketch $h(x) = f(x) - g(x)$

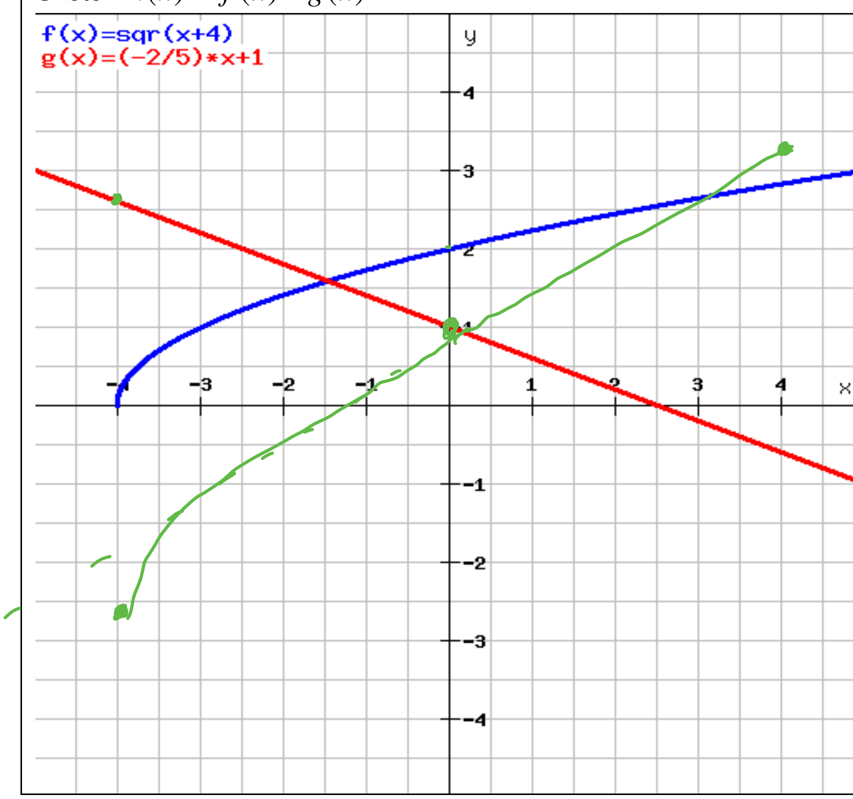


Table of Values:

x	$f(x)-g(x)$
-4	$0-2.6=-2.6$
0	$2-1=1$
4	$2.7-(-.6)=3.3$

Domain:

$\{x \mid x \geq -4,\ x \in \mathbb{R}\}$

Range:

$\{y \mid y \geq -2.6,\ y \in \mathbb{R}\}$

Sketch $h(x) = f(x) \bullet g(x)$

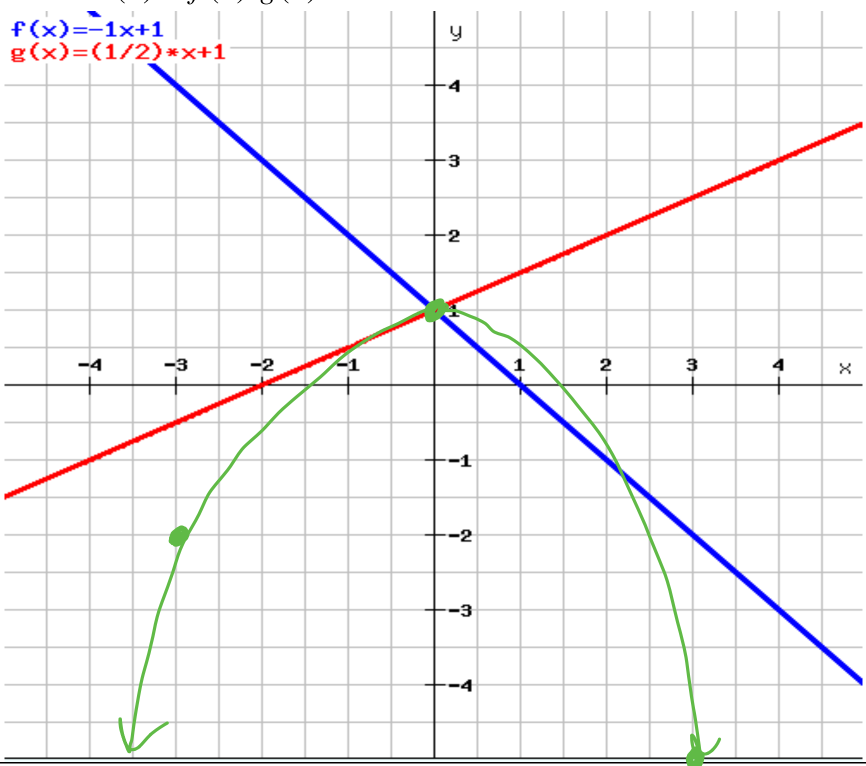


Table of Values:

x	$(f \circ g)(x)$
-3	$4(-\frac{1}{2}) = -2$
0	$1(1) = 1$
3	$-2(2.5) = -5$

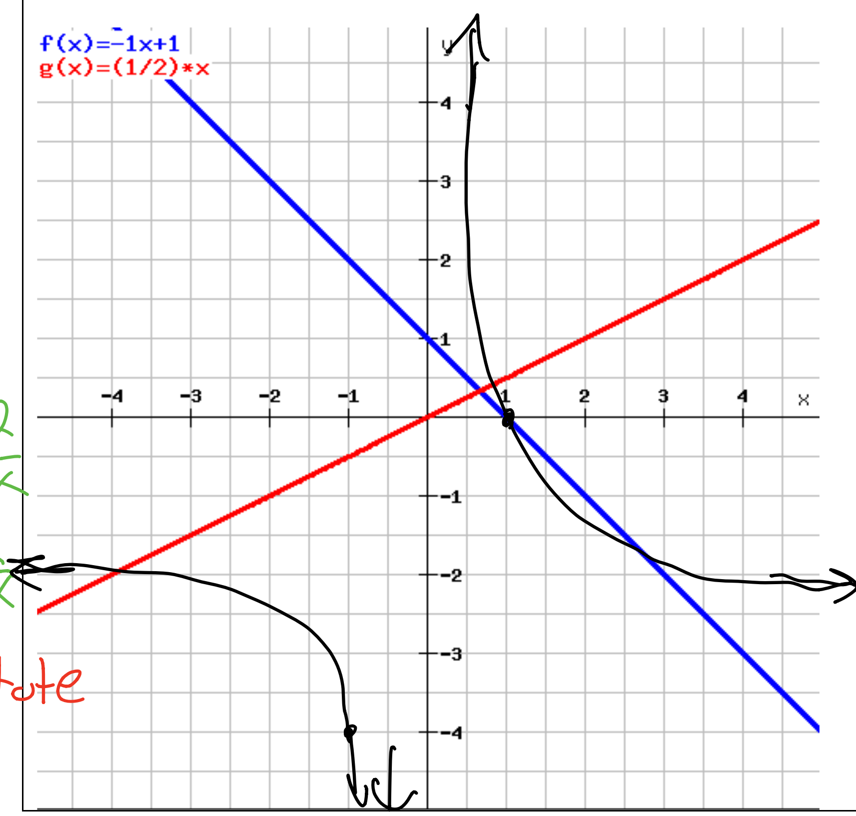
Domain:

$\{x \mid x \in \mathbb{R}\}$

Range:

$\{y \mid y \leq 1, y \in \mathbb{R}\}$

Sketch $h(x) = \dfrac{f(x)}{g(x)}$



$\dfrac{-x+1}{\frac{x}{2}}$

$= \dfrac{2(-x+1)}{x}$

$= \dfrac{-2x}{x} + \dfrac{2}{x}$

$= -2 + \dfrac{2}{x}$

asymptote @ $x = 0$

Table of Values:

x	
1	$\frac{0}{.5} = 0$
-1	$\frac{+2}{-.5} = -4$

Domain:

$\{x \mid x \neq 0, x \in \mathbb{R}\}$

Range:

$\{y \mid y \neq -2, y \in \mathbb{R}\}$