2.5 Composition of Functions

When two functions, $f(x)$ and $g(x)$, are combined after one function has been substituted into the other, a *composite function* has been created. The output of one function becomes the input of the other function.

For example, $f(g(x))$ means each "x" term in $f(x)$ is substituted by the function $g(x)$

Given $f(x)=5x$ and $g(x)=x^2+2x-3$, find

a. $f(g(2))$

$g(2) = (2)^2 + 2(2) - 3$
$= 4 + 4 - 3$
$= 5$

$f(5) = 5(5)$
$= 25$

b. $g(f(1))$

$f(1) = 5(1)$
$= 5$

$g(5) = 5^2 + 2(5) - 3$
$= 25 + 10 - 3$
$= 32$

hint: 32

Note: $f(g(x))$ is read as "f of g of x" and is equivalent to $(f \circ g)(x)$

Given $f(x)=4x+1$ and $g(x)=2-x$, find

a. $g(5)$

$g(5) = 2-5$
$= -3$

b. $f(g(5))$

$f(-3)$
$= 4(-3) + 1$
$= -11$

c. $(f \circ g)(x) = f(g(x))$

$f(x) = 4x + 1$
$(f \circ g)(x) =$
$4(2-x) + 1$

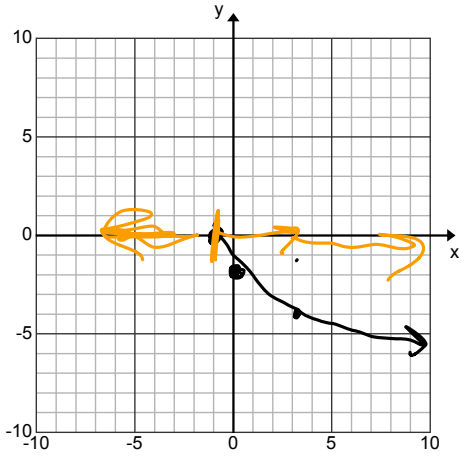
Graphing composite functions

Given that $f(x) = -2x$, and $g(x) = \sqrt{x+1}$

Determine $y = f(g(x))$

$= -2(\sqrt{x+1})$

b. Graph the function $y = f(g(x))$



	State Domain	State Range
$y = f(x)$	$x \in \mathbb{R}$	$y \in \mathbb{R}$
$y = g(x)$	$x \geq -1$	$y \geq 0$
$y = f(g(x))$	$x \geq -1$	$y \leq 0$

Composition of functions with formulas

The surface area (SA) of a sphere is a function of radius r, given as $SA(r) = 4\pi r^2$.

Express the area as a function of diameter d

$2r = d$

$r = \frac{d}{2}$

$r(d) = \frac{d}{2}$

$SA(r(d)) = 4\pi\left(\frac{d}{2}\right)^2$

$= \frac{4\pi d^2}{4}$

$= \pi d^2$

Calculate the surface area of the sphere when the diameter is 2.5 cm.

$SA(r(2.5))$

$= \pi(2.5)^2$

$\approx 19.6 \text{ cm}^2$