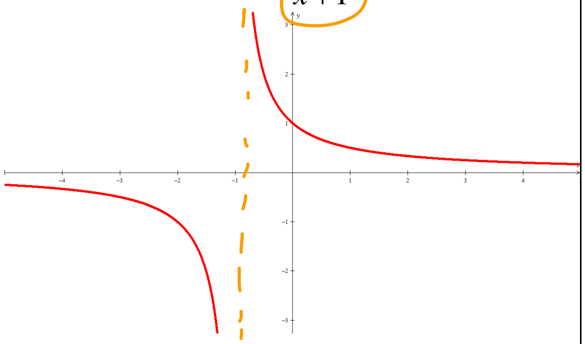
2.6 Rational Functions

Definition: A rational function is any function of the form: $R(x)=\dfrac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions. This is very similar to the idea of a rational number is the division of 2 integers. When graphing rational functions there are 4 different types of "problems" can occur on their graph.

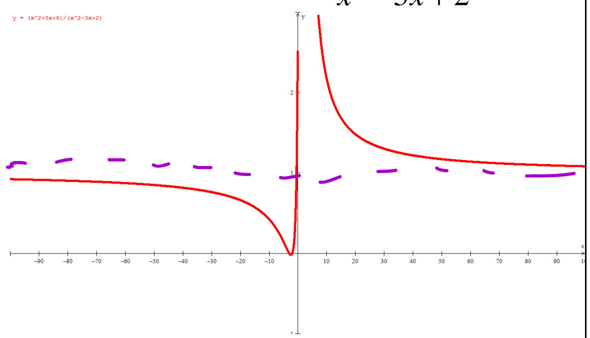
Vertical Asymptote $y=\dfrac{1}{x+1}$



Location of Asymptote $x=-1$

Domain $\{x \mid x\neq -1,\ x\in\mathbb{R}\}$

Horizontal Asymptote: $y=\dfrac{x^2+5x+6}{x^2-3x+2}$

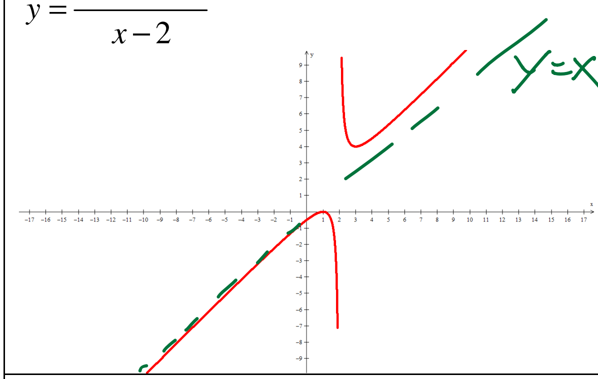


Location of Asymptote

$y=1$

Oblique (Slant Asymptote)

$y=\dfrac{x^2-2x+1}{x-2}$



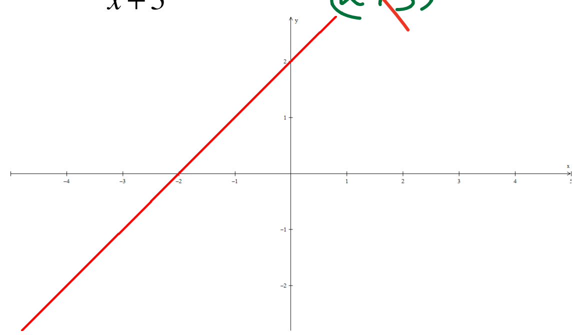
$y=x$

Location of asymptote: $y=x$

Domain: $\{x \mid x\neq 2,\ x\in\mathbb{R}\}$

Hole (Really hard to show on a graph)

$y=\dfrac{x^2+5x+6}{x+3}=\dfrac{\cancel{(x+3)}(x+2)}{\cancel{(x+3)}}$

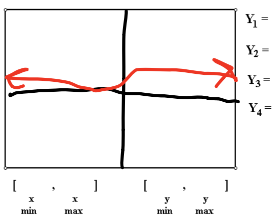
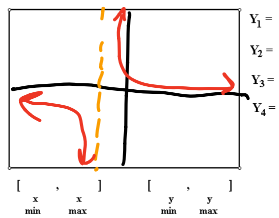
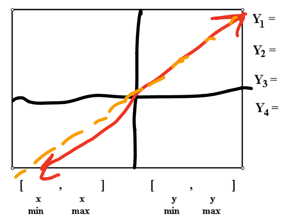


Location of hole:

Domain:

1. Horizontal Asymptotes. Horizontal asymptotes describe the "end behaviour" of a function. That is to say what happens to the graph as x becomes large and positive or large and negative.

Graph the following functions on your graphing calculator, and describe what is happening to the functions *y values* as x becomes large.

$y=\dfrac{(x^2+5x+6)}{(2x^2+6x+9)}$	$y=\dfrac{(x^2+3x+7)}{(x^3+5x+2)}$	$y=\dfrac{(x^3+7x+2)}{(x^2+4x+8)}$
		
Horizontal asymptote? @ $y=\frac{1}{2}$	Horizontal Asymptote? @ $y=0$	Horizontal Asymptote? $y=x$

degree How can you tell if a function has a horizontal asymptote with out graphing it?

horizontal: num = den, divide the leading coefficients | vertical: divide by zero | slant: num = den + 1

Ex. What are the horizontal asymptotes of:

$y=\dfrac{3x^3+5x-9}{5x^3-2x-10}$ $y=\frac{3}{5}$	$y=\dfrac{5x-3x^2-8000}{2x^3+5x-1}$ $y=0$	$y=\dfrac{4x^5-3x^2}{x^4-12}$ $y=x$

2. If a function doesn't have a horizontal asymptote, it might have an oblique (slant) asymptote. Linear oblique asymptotes occur when the degree of the numerator is one greater than the degree of the denominator.

Ex. Which of the following functions have oblique asymptotes?

$y=\dfrac{x^2+2}{x-1}$	$y=\dfrac{x^2-3x+5}{2-x^2}$	$y=\dfrac{x^3+2x-3}{2x-5x^2}$

$2=1+1$ $2=2$ $3=2+1$

$\therefore$ oblique $\therefore$ horizontal @ $y=-1$ $\therefore$ oblique

Find the oblique asymptotes of: $y=\dfrac{x^2-2x-8}{x-3}$

$$x-3\overline{)x^2-2x-8}$$

$x+1$

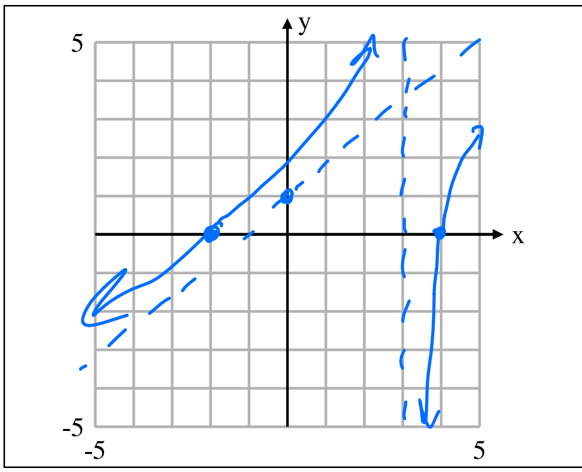
x^2-3x

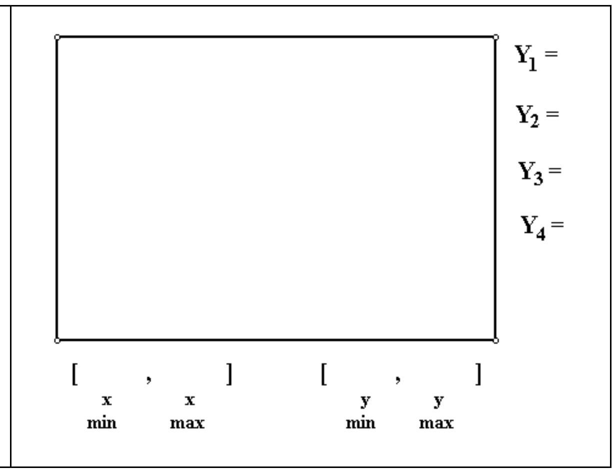
$x-8$

$x-3$

-5

$= x+1 - \dfrac{5}{x-3}$





3. <u>Vertical Asymptotes/Holes</u>. Rational functions whose denominators that have real roots will have either vertical asymptotes or holes. Holes will occur in a graph when a factor in the denominator cancels with a factor in the numerator. Vertical asymptotes will occur when the factor in the denominator doesn't cancel.

Example: Determine if the function has a hole (H) or a vertical asymptote (VA) and state its location (x-coordinate only)

$y=\dfrac{x+2}{x-3}$	$y=\dfrac{(x+2)(x+3)}{(x-3)(x-1)}$
$y=\dfrac{(x+2)(x+3)}{(x+2)}$	$y=\dfrac{(x+2)(x+3)}{(x-1)(x+3)}$
$y=\dfrac{(x+2)(x-3)}{(x+2)(x-3)(x+1)}$	$y=\dfrac{x^2+5x+6}{x+2}$

If a function has a vertical asymptote, we take note of it and make a domain statement that doesn't allow this value to be substituted in to the function.

Day 1