3.3 Laws of Logarithms

Exponent Rule	Logarithmic Rule
Addition of Exponents: $x^m \times x^n = x^{m+n}$ Example: $5^2(5) = 5^{2+1}$	Addition of Logarithms: If $x, y > 0$ and $a > 0, a \neq 1$, then $\log_a(x \times y) = \log_a x + \log_a y$ Example: $\log(10(100)) = \log(10) + \log(100)$
Subtraction of Exponents $\frac{x^m}{x^n} = x^{m-n}$ Example: $\frac{10^3}{10^2} = 10^{3-2}$	Subtraction of Logarithms: If $x, y > 0$ and $a > 0, a \neq 1$, then $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ Example: $\log\left(\frac{100}{10}\right) = \log(100) - \log(10)$
Exponent of Zero: $x^0 = 1$ Example: $\pi^0 = 1$	Logarithms of 1 and *a*: $\log_a 1 = 0$ and $\log_a a = 1$ Example: $\log_\pi(\pi) = 1$
	Power Rule for Logarithms: If $x, n \in \Re$, and $x > 0$, then $\log_a x^n = n \log_a x$ Example: $\log(2^x) = x\log(2)$
	Change of Base Rule for Logarithms: If $x > 0$ and $a, b > 0, a, b \neq 1$, then $\log_a x = \frac{\log_b x}{\log_b a}$ Example: $\log_7(42) = \frac{\log_{10}(42)}{\log_{10}(7)}$
Fractional Exponent Rule for Exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ Example: $2^{\frac{3}{4}} = \sqrt[4]{2^3}$	
$x^{-m} = \frac{1}{x^m}$ Example: $5^{-1} = \frac{1}{5}$	

<u>NOTE:</u> Logarithms are an operation (like square root), you CANNOT DISTRIBUTE IT!

$\log_a(x+y) \neq \log_a x + \log_a y$

Working with the rules, Simplify:

$\log_2 5 + \log_2 3$ adding same base

$= \log_2(5(3))$

$= \log_2(15)$

$\log_6 9 + \log_6 4$

$= \log_6(9(4))$

$= \log_6(36)$

$\log_3 45 - \log_3 5$

$= \log_3\left(\frac{45}{5}\right)$

$= \log_3(9)$

$\log 20 + \log 5$

$= \log(20(5))$

$= \log(100)$

$= 2$

Write as a single logarithm

$\log A + 2\log B - 3\log C$

$= \log A + \log B^2 - \log C^3$

$= \log\left(\frac{AB^2}{C^3}\right)$

$\log A - 4\log C - 5\log D$

$= \log A - \log C^4 - \log D^5$

$= \log\left(\frac{A}{C^4 D^5}\right)$

Write in terms of log a, log b, log c

$\log\left(\frac{a^2b^3}{c^4}\right)$

$= \log A^2 + \log B^3 - \log C^4$

$= 2\log A + 3\log B - 4\log C$

$\log\left(\frac{100a^3}{b^4\sqrt[3]{c^2}}\right)$

$= \log(100) + \log A^3 - \log b^4 - \log C^{\frac{2}{3}}$

$= 2 + 3\log A - 4\log b - \frac{2}{3}\log C$

Estimate, and then evaluate to 4 decimal places (using the Change of Base Rule):

$\log_2 7$ ≈ <3 $= \frac{\log 7}{\log 2} \approx 2.8074$	$\log_7 55$ ≈ >2 $\frac{\log 55}{\log 7} \approx 2.0594$

Solve: $8^{2x+3} = 4^{x-1}$

Hint:
if the bases are different
how can you apply the rules?

$(2^3)^{2x+3} = (2^2)^{x-1}$

$2^{(3)(2x+3)} = 2^{2(x-1)}$

$6x + 9 = 2x - 2$

$6x - 2x = -2 - 9$

$4x = -11$

$x = -\frac{11}{4}$

Higher Level Thinking:
If $\log 4 = x$ and $\log 5 = y$, what is $\log 50$ in terms of *x* and *y*.

$= \log 5 + \log 5 + \log 4 - \log 2$ (y, y, x)

$= \frac{\log[5(5)(4)]}{\log 2} = \log 50$

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