3.4 Applications of Logarithms

$A = P\left(1+\frac{i}{n}\right)^{t\times n}$	$A = P\left(X\right)^{\frac{t}{n}}$

Ex. 1
Alex invests $1200 paying 8% interest p.a. compounded semi-annually. Determine how long it takes his investment to double.

$2400 = 1200\left(1+\frac{.08}{2}\right)^{2t}$

$2 = \left(1+\frac{.08}{2}\right)^{2t}$

$2 = (1.04)^{2t}$

$2 = \left[(1.04)^2\right]^t$

$\log(2) = \log(1.0816^t)$

$\log(2) = t\log(1.0816)$

$\frac{\log(2)}{\log(1.0816)} = t$

8.84 years $= t$

Ex. 2
Scopenium has a half life of 35 days. If there is currently a sample of 80 kg presently, how long does it take until there is only 2 kg present?

$2 = 80\left(\frac{1}{2}\right)^{\frac{t}{35}}$

$\frac{1}{40} = \left[\left(\frac{1}{2}\right)^{\frac{1}{35}}\right]^t$

$\log\left(\frac{1}{40}\right) = \log(.980^t)$

$\log\left(\frac{1}{40}\right) = t\log(.980)$

$\frac{\log\left(\frac{1}{40}\right)}{\log(.980)} = t$

182.6 days $= t$

Ex. 3
If there are currently 200 bacteria living in Darren's locker, and in 5 days there are 4000. What is the doubling time of the bacteria?

$4000 = 200(2)^{\frac{5}{n}}$

$20 = (2^5)^{\frac{1}{n}}$

$\log(20) = \log(32^{\frac{1}{n}})$

$\log(20) = \frac{1}{n}\log(32)$

$n = \frac{\log(32)}{\log(20)}$

$n = 1.16$

Ex. 4
The intensity of light decreases by 3% for every 5 meters it travels into water. At the surface of the water a source of light is 3000 lumens. How far did the light travel if it is now 1000 lumens?

$1000 = 3000\left(1+\frac{-.03}{5}\right)^{5t}$

$\frac{1}{3} = (.994)^{5t}$

$\log\left(\frac{1}{3}\right) = \log(.970^t)$

$\log\left(\frac{1}{3}\right) = t\log(.970)$

$\frac{\log\left(\frac{1}{3}\right)}{\log(.970)} = t$

37m $= t$