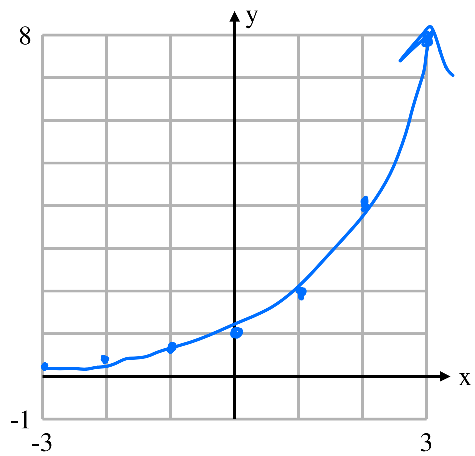
## 3.7 Graphing Exponential Functions

Graph: $y = 2^x$

| X | Y |
|---|---|
| -3 | $\frac{1}{8}$ |
| -2 | $\frac{1}{4}$ |
| -1 | $\frac{1}{2}$ |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |



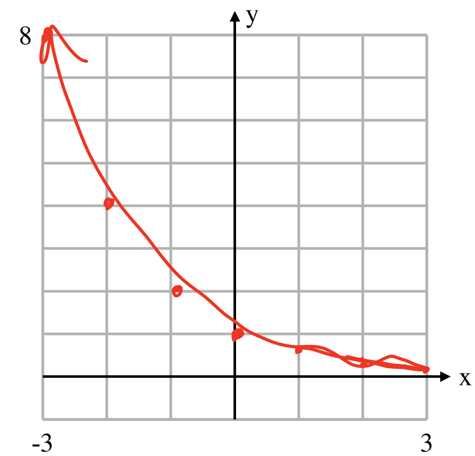

Domain:
$\{x \mid x \in \mathbb{R}\}$

Range
$\{y \mid y > 0, y \in \mathbb{R}\}$

Asymptote
horizontal @ $y = 0$

Graph: $y = \left(\frac{1}{2}\right)^x$

| X | Y |
|---|---|
| -3 | 8 |
| -2 | 4 |
| -1 | 2 |
| 0 | 1 |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{4}$ |
| 3 | $\frac{1}{8}$ |




Domain:
$\{x \mid x \in \mathbb{R}\}$

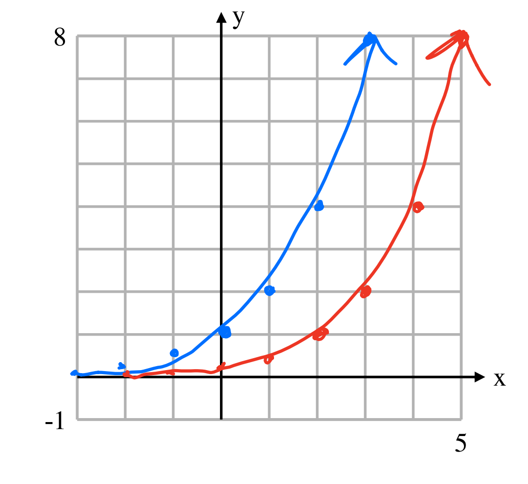
Range
$\{y \mid y > 0, y \in \mathbb{R}\}$

Asymptote
horizontal @ $y = 0$

What is the relationship between $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$?

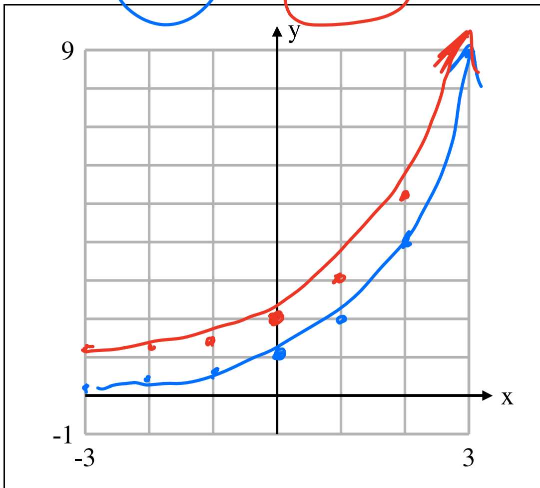
Increasing / Decreasing exponential $f^n$.

Graph: $y = 2^x$ and $y = 2^{x-2}$ on the same grid.




| $y = 2^x$ | $y = 2^{x-2}$ |
|---|---|
| Domain<br>$\{x \mid x \in \mathbb{R}\}$ | Domain<br>$\{x \mid x \in \mathbb{R}\}$ |
| Range<br>$\{y \mid y > 0, y \in \mathbb{R}$ | Range<br>$\{y \mid y > 0, y \in \mathbb{R}\}$ |
| Asymptote<br>horizontal @ $y = 0$ | Asymptote<br>horizontal @ $y = 0$ |

Graph: $y = 2^x$ and $y = 2^x + 1$ on the same grid.




| $y = 2^x$ | $y = 2^x + 1$ |
|---|---|
| Domain<br>$\{x \mid x \in \mathbb{R}\}$ | Domain<br>$\{x \mid x \in \mathbb{R}\}$ |
| Range<br>$\{y \mid y > 0, y \in \mathbb{R}\}$ | Range<br>$\{y \mid y > 1, y \in \mathbb{R}\}$ |
| Asymptote<br>horizontal @ $y = 0$ | Asymptote<br>horizontal $y = 1$ |

When transforming graphs, only the transformation up/down changes the location of the asymptote and range.

Without graphing, determine the range & asymptote of:

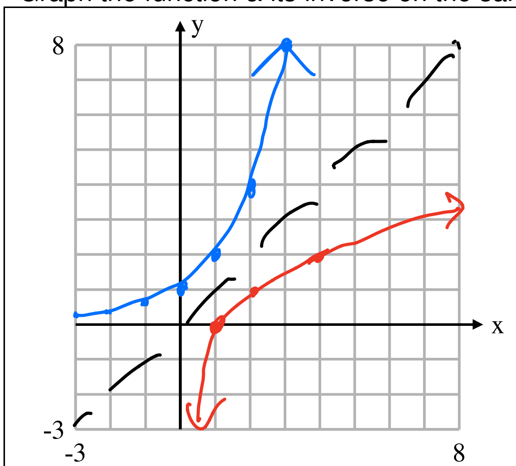
| Function | Range | Asymptote |
|---|---|---|
| $y = 3^x + 4$ | $y > 4$ | $y = 4$ |
| $y = 4^{x-3} + 2$ | $y > 2$ | $y = 2$ |
| $y = \left(\frac{1}{2}\right)^{x-1} - 4$ | $y > -4$ | $y = -4$ |
| $y = -\left(2^{x-4}\right) + 5$ | $y < 5$ | $y = 5$ |

The <u>inverse</u> of any *exponential function* is a *logarithmic function*. In mathematics terms, $y = a^x$ is an exponential function, and it's inverse is $y = \log_a x$.

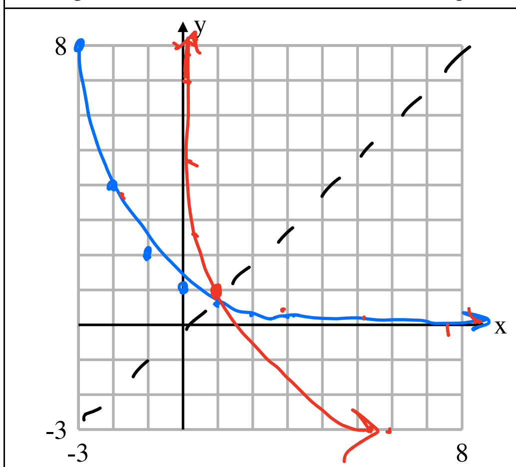
Given the function, find it's inverse:

| Function | Inverse |
|---|---|
| $y = 3^x$ | $\log_3(x)$ |
| $y = 5^x$ | $\log_5(x)$ |
| $y = 8^x$ | $\log_8(x)$ |
| $y = \log_4 x$ | $4^x$ |
| $y = \log_9 x$ | $9^x$ |

Graph the function & its inverse on the same grid:




| Function: $y = 2^x$ | Inverse: $y = \log_2 x$ |
|---|---|
| Domain<br>$\{x \mid x \in \mathbb{R}\}$ | Domain<br>$\{x \mid x > 0, x \in \mathbb{R}\}$ |
| Range<br>$\{y \mid y > 0, y \in \mathbb{R}\}$ | Range<br>$\{y \mid y \in \mathbb{R}\}$ |
| Asymptote<br>horizontal @ $y = 0$ | Asymptote<br>vertical @ $x = 1$ |




| Function: $y = \left(\frac{1}{2}\right)^x$ | Inverse: $y = \log_{\frac{1}{2}} x$ |
|---|---|
| Domain<br>$\{x \mid x \in \mathbb{R}\}$ | Domain<br>$\{x \mid x > 0, x \in \mathbb{R}\}$ |
| Range<br>$\{y \mid y > 0, y \in \mathbb{R}\}$ | Range<br>$\{y \mid y \in \mathbb{R}\}$ |
| Asymptote<br>horizontal @ $y = 0$ | Asymptote<br>vertical @ $x = 0$ |

Finding the inverse (algebraically):

| | |
|---|---|
| $y = 2^{x-1} + 4$ | $y = 3^{x+2} - 6$ |
| $x = 2^{y-1} + 4$ | $x = 3^{y+2} - 6$ |
| $x - 4 = \frac{2^{y}}{2}$ | $x + 6 = 3^{y} \cdot 3^{2}$ |
| $2(x-4) = 2^{y}$ | $\frac{x+6}{3^{2}} = 3^{y}$ |
| $\log_2[2(x-4)] = y \log_2(2)$ | $\log_3\left(\frac{x+6}{3^{2}}\right) = y \log_3(3)$ |
| $\log_2(2) + \log_2(x-4) = y$ | $\log_3(x+6) - \log_3(3^{2}) = y$ |
| $1 + \log_2(x-4) = y$ | $\log_3(x+6) - 2 = y$ |
| $y = \log_2(x-4) + 1$ | $y = \log_3(x+6) - 2$ |