3.9 Solving Logarithmic Equations

Recall: $\log_a a^x = x \log_a a$
$= x \times 1$

Add the following rule:
$a^{\log_a x} = x$

Verify the rule by calculating:

$10^{\log 5}$ $= 5$	$10^{\log 7}$ $= 7$	$10^{\log 1.265}$ $= 1.265$

Using the "new" rule

Simplify:

$2^{\log_2 5x + \log_2 3x}$ $2^{\log_2(15x^2)}$ $= 15x^2$	$a^{\log_a 9x - \log_a 3}$ $a^{\log_a(3x)}$	$16^{\log_2 3x}$ $2^{4(\log_2(3x))} = 2^{\log_2(3x)^4}$ $= (3x)^4$

When solving logarithmic equations, the goal is to make one of the following situations occur.

1. Have a single log on one side of the equation (or nested logs) equaling a number
2. Have a single log on one side of the equation (or nested logs) equaling a log with the same base
    a. If the bases are not the same… you will have to change the base so that they are the same.

Solve algebraically:

$\log_2(x-3) + \log_2(x+1) = 5$ $\log_2[(x-3)(x+1)] = 5$ $2^5 = (x-3)(x+1)$ $32 = x^2 + x - 3x - 3$ $0 = x^2 - 2x - 35$ $= (x-7)(x+5)$ $x = 7$ or -5	$\log(6-x) - 2\log x = 0$ $\log \frac{6-x}{x^2} = 0$ $10^0 = \frac{6-x}{x^2}$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3$ or 2

$\log(x+2) = 1 - \log(x-1)$

$\log(x+2) + \log(x-1) = 1$

$\log[(x+2)(x-1)] = 1$

$10^1 = (x+2)(x-1)$

$10 = x^2 - x + 2x - 2$

$0 = x^2 + x - 12$

$0 = (x-3)(x+4)$

$x = 3$ or $\cancel{-4}$

$5\log_2 m - 3\log_2 m = 4$

$\log_2\left(\frac{m^5}{m^3}\right) = 4$

$2^4 = m^2$

$\pm 4 = m$

$\log_2(\log_3 x) = 0$

$\log_2\left(\frac{\log_2 x}{\log_2 3}\right) = 0$

$1 = \frac{\log_2 x}{\log_2 3}$

$\rightarrow \log_2 3 = \log_2 x$

$3 = x$

$\log_9(\log_2 x) = -\frac{1}{2}$

$\log_9\left(\frac{\log_9 x}{\log_9 2}\right) = -\frac{1}{2}$

$\frac{1}{3} = \frac{\log_9 x}{\log_9 2}$

$\rightarrow \frac{1}{3}\log_9 2 = \log_9 x$

$\log_9 2^{\frac{1}{3}} = \log_9 x$

$2^{\frac{1}{3}} = x$

$(\log_x 5)(\log_5 81) = 2$

$\cancel{\log_x 5}\left(\frac{\log_x 81}{\cancel{\log_x 5}}\right) = 2$

$x^2 = 81$

$x = \pm 9$

$2^{\log_3 x} = \frac{1}{8}$

$2^{\frac{\log_2 x}{\log_2 3}} = 2^{-3}$

$\frac{\log_2 x}{\log_2 3} = -3$

$\log_2 x = -3\log_2 3$

$\rightarrow \log_2 x = \log_2 3^{-3}$ ☆

$x = 3^{-3}$

$x = \frac{1}{27}$

$\log_{\frac{1}{2}} 5 = \log_2 t^2 - \log_2 5t$ ☆

$-\log_2 5 = \log_2\left(\frac{t}{5}\right)$

$0 = \log_2\left(\frac{t}{5}\right) + \log_2 5$

$0 = \log_2\left(\frac{\cancel{5}t}{\cancel{5}}\right)$

$0 = \log_2 t$

$t = 1$