5.3 Graphing $y = a\sin\left(b\left(x-c\right)\right)+d$ and $y = a\cos\left(b\left(x-c\right)\right)+d$

a -- Amplitude
b – Period change (chapter 1 – horizontal expansion/compression)
c – Phase shift
d – Vertical displacement

In chapter 1, we graphed vertical expansion/compressions, next, horizontal expansion/compressions, and then translation left/right, and finally up/down.

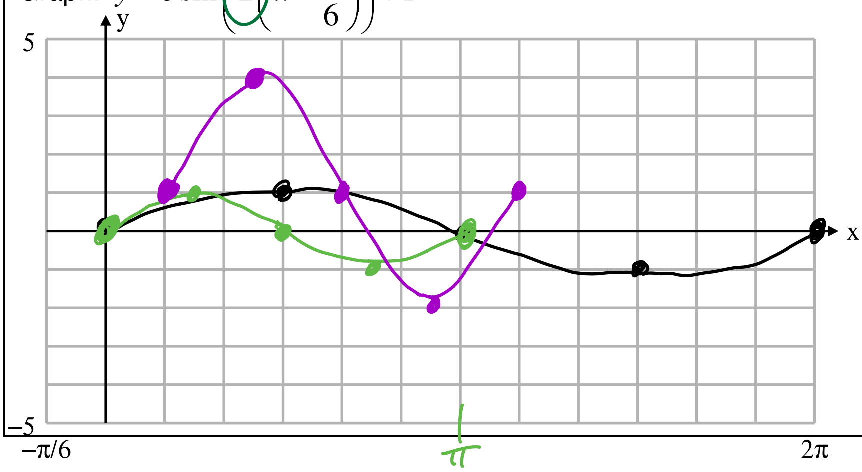
When graphing in chapter 5 we can graph in a slightly different order to speed up the process. We can do this because of the repetitive(periodic) function as well as the range of the initial function is always $-1 \le y \le 1$. The order for chapter 5:

1. Graph vertical displacement & amplitude at the same time
2. Graph the period change
3. Phase shift the graph

1) normal
2) period change $\left(\frac{2\pi}{6}\right)$
3) translations

For example:

Graph: $y = 3\sin\left(2\left(x-\frac{\pi}{6}\right)\right)+1$



Amplitude: 3

Vertical Displacement: 1

Phase Shift: $\frac{\pi}{6}$

Period: $\frac{2\pi}{2} = \pi$

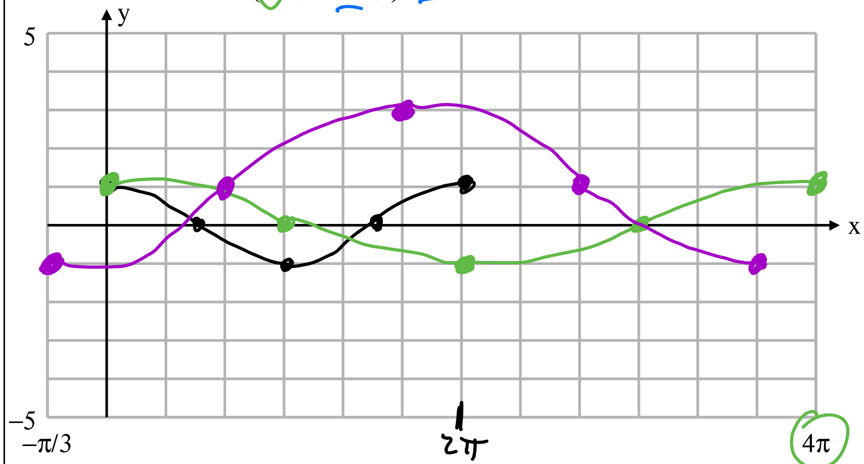
Maximum: $y = 4$

Minimum: $y = -2$

NOTE: The scale of the graph is important. Count the number of "squares" from 0 to 2π to determine the scale of the graph.

period $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$

Graph: $y = -2\cos\left(\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right)+1$



Amplitude: 2

Vertical Displacement: 1

Phase Shift: $-\frac{\pi}{3}$

Period: 4π

Maximum: $y = 3$

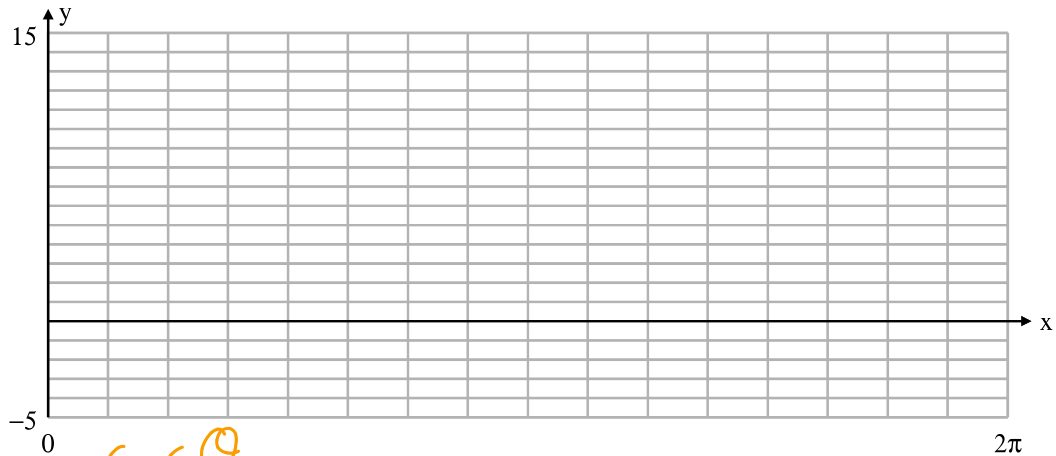
Minimum: $y = -1$

Determine the following properties of the function without graphing:

$y = 7\cos\left(\frac{2}{3}\left(x-\frac{\pi}{6}\right)\right)-4$	$y = -7\sin\left(\frac{9}{2}\left(x-\frac{\pi}{2}\right)\right)+2$
Amplitude: 7	Amplitude: 7
Vertical Displacement: -4	Vertical Displacement: 2
Phase Shift: $\frac{\pi}{6}$	Phase Shift: $\frac{\pi}{2}$
Period: $\frac{2\pi}{\frac{2}{3}} = \frac{6\pi}{2} = 3\pi$	Period: $\frac{4\pi}{9}$
Maximum: $y = 3$	Maximum: $y = 9$
Minimum: $y = -11$	Minimum: $y = -5$

Fast way to calculate the period: $period = \frac{2\pi}{b}$ … but we can also use the ideas from chapter 1 (thinking it as a horizontal compression/expansion).

Example: A sinusoidal function has a maximum at $\left(\frac{\pi}{8}, 12\right)$, the next minimum is at $\left(\frac{3\pi}{4}, -4\right)$. Determine a sinusoidal function that best represents this situation (hint use cosine…)



$\cos\theta$

1) 16 units of height

$\therefore \; a = 8$

2) max $\frac{\pi}{8} \rightarrow \frac{3\pi}{4}$ $\quad \frac{\pi}{8} \rightarrow \frac{6\pi}{8}$

$$\frac{2\pi}{b} = \frac{16\pi}{8}$$

$$\frac{2\pi}{10\pi}(8) = b$$

$$\frac{8}{5} = b$$

3) max @ $\frac{\pi}{8}$

$$y = a\cos b(x - c) + d$$

$$= 8\cos \frac{8}{5}\left(x - \frac{\pi}{8}\right) + 4$$