Chapter 6 – Trigonometric Equations and Identities
6.1 Introduction to Proof & Reciprocal Identities

A proof {trigonometric one} requires you to manipulate one side {or both} of an algebraic equation so that they ultimately match. What you need to do for a proof is to show me that the left hand side of the equation is exactly the same of the right hand side of the equation.

This is done by:

1. Finding common denominators to add fractions
2. Adding\Subtracting like terms together in an expression
3. Distributing through a set of brackets
4. Factoring common terms
5. Changing Division of Two Fractions into Multiplication
6. Canceling Common Terms
7. Using the "conjugate" to write a fraction in a different form.
8. Substituting Existing Identities

YOU ARE NOT ALLOWED TO DO – EVER – TO MOVE "STUFF" FROM ONE SIDE OF AN EQUATION TO THE OTHER! YOU ARE NOT ALLOWED TO "SOLVE" – You can only manipulate how "stuff" looks.

An Identity is a statement that we have either:

a. assumed it to be true (a definition)
b. or have already proved it to be true

Chapter 4 and 5 has some identities that we have used:

$\sec\theta = \dfrac{1}{\cos\theta}$	$\csc\theta = \dfrac{1}{\sin\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	

The "three" main types of problems for this section are:

a. Simplify: you attempt to change from the given expression to a much smaller one
b. $\boxed{\text{Verify:}}$ you graph both sides of an expression to show that they are the same, or you substitute a known value for θ into both sides of the equation and show that they are the same. This is weak evidence because you must show that it works for all of the values of θ in the domain.
c. Prove: Show that one side is exactly the same as the "other side"

Example: Simplify:

$\sec^2 x \times \cos x$ $\frac{1}{\cos^2 x} \cdot \cos x = \frac{\cos x}{\cos^2 x}$ $= \frac{1}{\cos x}$ $= \sec x$	$\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}$ $= \sin x$

Example: Verify

$\sin\theta \times \cos\theta \times \tan\theta = \cos\theta$	$\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

$\sin\theta \, \cancel{\cos}\theta \, \frac{\sin\theta}{\cancel{\cos}\theta} = \cos\theta$

$\sin^2\theta = \cos\theta$

$\sin\theta \, \frac{\sin\theta}{\cos\theta} + \cancel{\sin}\theta \, \frac{\cos\theta}{\cancel{\sin}\theta} = \sec\theta$

$\frac{\sin^2\theta}{\cos\theta} \cos\theta + \cos\theta \cos\theta = \sec\theta \cos\theta$

$\sin^2\theta + \cos^2\theta = 1$

Example: Prove:

$\frac{\cot\theta}{\csc\theta} = \cos\theta$	$\sin\theta\cot\theta = \cos\theta$
$\cos\theta(\sec\theta - 1) = 1 - \cos\theta$	$\frac{1-\tan\theta}{1-\cot\theta} = -\tan\theta$

$\frac{\cot\theta}{\csc\theta} = \cos\theta$

LHS	RHS
$\frac{\cot}{\csc} = \frac{\frac{\cos}{\sin}}{\frac{1}{\sin}}$	$\cos$
$= \frac{\cos}{\cancel{\sin}} \cdot \frac{\cancel{\sin}}{1}$	
$= \cos$	

$\sin\theta\cot\theta = \cos\theta$

LHS	RHS
$\cancel{\sin} \cdot \frac{\cos}{\cancel{\sin}}$	$\cos$
$= \cos$	

$\cos\theta(\sec\theta - 1) = 1 - \cos\theta$

LHS	RHS
$\frac{\cos}{\cos} - \cos$	$1 - \cos$
$1 - \cos$	

$\frac{1-\tan\theta}{1-\cot\theta} = -\tan\theta$

LHS	RHS
$\frac{1 - \frac{\sin}{\cos}}{1 - \frac{\cos}{\sin}} = \frac{\frac{\cos - \sin}{\cos}}{\frac{\sin - \cos}{\sin}}$	$-\tan$
$\frac{(\cos - \sin)}{\cos} \cdot \frac{\sin}{(\sin - \cos)}$	
$\frac{-(\cancel{-\cos + \sin})\sin}{\cancel{(\sin - \cos)}\cos}$	
$-\tan$	