6.3 Sum & Difference Identities

Since sine, cosine, and tangent are "operators," that is to say that they do "something" to the angle inside the brackets. You do not get to distribute the trigonometric function.

I.e. $\sin(30+25) \neq \sin 30 + \sin 25$

Instead, sine, cosine, and tangent of sums/differences of angles can be written as sums/differences in a much different way (as shown below).

$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\sin(A-B)=\sin A\cos B-\cos A\sin B$	$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$	$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

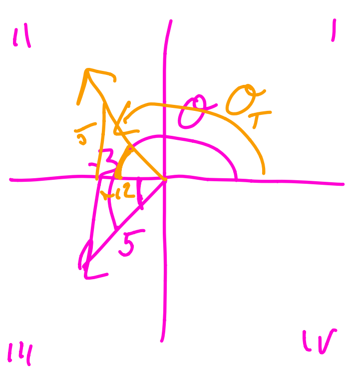
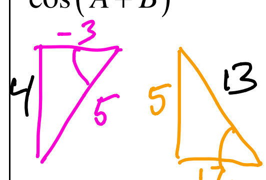
We can use these identities to:

a. Evaluate special angles that are not on the special triangle.

$\sin\left(\frac{\pi}{12}\right)=\sin\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ $=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$ $=\frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}-\frac{1}{2}\,\frac{\sqrt{2}}{2}$ $=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$ $=\frac{\sqrt{6}-\sqrt{2}}{4}$	$\tan\left(\frac{\pi}{12}\right)=\tan\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)=\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ $=\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ $=\dfrac{\sqrt{3}-1}{1+\sqrt{3}}\left(\dfrac{1-\sqrt{3}}{1-\sqrt{3}}\right)\Rightarrow\dfrac{\sqrt{3}-3-1+\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3}$ $=\dfrac{2\sqrt{3}-4}{-2}=-(\sqrt{3}-2)$ $=2-\sqrt{3}$

b. Find expressions for sums/differences of arbitrary angles A and B.

Ex. If $\cos A=-\frac{3}{5}$ and A is in quadrant III, and $\tan B=-\frac{5}{12}$ and B is in quadrant II, find the value of

	$\cos(A+B)$	$\tan(A+B)$
	 $\left(-\frac{3}{5}\right)\left(\frac{-12}{13}\right)-\left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$ $\frac{36}{65}-\frac{20}{65}$ $=\frac{16}{65}$	$\dfrac{\frac{4}{-3}+\frac{-5}{12}}{1-\frac{4}{-3}\left(\frac{-5}{12}\right)}=\dfrac{-\frac{7}{4}}{\frac{4}{9}}$ $=-\frac{63}{16}$

c. "Simplify" expressions.

Ex. Write as a single trigonometric function: $\cos 6x \cos 2x + \sin 6x \sin 2x$

$= \cos(6x - 2x)$

$= \cos(4x)$

Ex. Simplify:

$\cos(\pi - x)$	$\tan(\pi + x)$
$= \cos\pi \cos x + \sin\pi \sin x$ $= -1 \cos x + 0 \sin x$ $= -\cos x$	$= \dfrac{\tan\pi + \tan x}{1 + \tan\pi \tan x}$ $= \dfrac{0 + \tan x}{1 + 0} = \tan x$

d. Prove equations:

Ex. Prove: $\sin\left(\frac{\pi}{4} + \theta\right) + \sin\left(\frac{\pi}{4} - \theta\right) = \sqrt{2}\cos\theta$

$= \sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta$

$= \frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\sin\theta + \frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta$

$= \frac{\sqrt{2}}{2}(\cos\theta + \sin\theta + \cos\theta - \sin\theta)$

$= \frac{\sqrt{2}}{2}(2\cos\theta)$

$= \sqrt{2}\cos\theta$