Complete the square

Monday, March 07, 2016 12:29 PM

Completing the Square

We have seen the advantages of having our quadratic equation in vertex form.

$$f(x) = a(x-p)^2 + q$$

This is very nice for graphing as we can find the shifts, stretches, and reflection very easily.

Completing the square is a useful technique for getting the vertex form of the equation when you begin with the standard form.

$$f(x) = Ax^2 + Bx + C$$

Example 1:

Complete the square for:

$$y = x^2 - 8x + 5$$

$A=1 \quad B=-8 \quad C=5$

Here's how to do it:

1. Group the x-terms together.
2. Divide the 'x' coefficient by 2, Then square it. ie: $\left(\frac{B}{2}\right)^2$
3. Add and subtract that value to your equation.
4. The result will be a perfect square. ie: easy factoring.

$\left[\frac{-8}{2}\right]^2 = (-4)^2 = 16$

$$y = (x^2 - 8x) + 5$$
$$= (x^2 - 8x + 16 - 16) + 5$$
$$= (x-4)(x-4) - 16 + 5$$
$$= (x-4)^2 - 11$$

$y = (x^2 + 6x) + 5$

$\left(\frac{3}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$y = (x^2 + 6x + 9 - 9) + 5$

$y = (x+3)(x+3) - 9 + 5$

$= (x+3)^2 - 4$

$a \cdot b = 9$

$a + b = 6$

3, 3

You try:

$f(x) = (x^2 - 10x) + 6$ ||| $y = (x^2 - 4x) - 3$

$y = (x^2 - 10x + 25 - 25) + 6$

$= (x-5)^2 - 25 + 6$

$= (x-5)^2 - 19$

$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$= \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$y = (x^2 - 4x + 4 - 4) - 3$

$= (x-2)^2 - 4 - 3$

$= (x-2)^2 - 7$

$(x-2)^2 = (x-2)(x-2)$

$x^2 - 2x - 2x + 4$

Re-write into vertex form:

aka: complete the square to get standard form into vertex form.

$$f(x) = (x^2 + 5x) - 2$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$f(x) = \left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) - 2$$

$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 2$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{8}{4}$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{33}{4}$$

$$-\frac{25}{4} - \left[\frac{2}{1} \cdot \frac{4}{4}\right]$$

$$-\frac{25}{4}$$

What if the coefficient in front of x^2 isn't equal to one?

Complete the square:

$$f(x) = 3x^2 - 12x - 9$$

$$f(x) = 3(1x^2 - 4x) - 9$$

$$= 3(x^2 - 4x + 4 - 4) - 9$$

$$= 3(x-2)^2 - 4(3) - 9$$

$$= 3(x-2)^2 - 12 - 9 \quad = 3(x-2)^2 - 21$$

$$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

$$y = -x^2 + 6x - 7$$

$$y = -(x^2 - 6x) - 7$$

$$y = -(x^2 - 6x + 9 - 9) - 7$$

$$y = -(x-3)^2 + 9 - 7$$

$$y = -(x-3)^2 + 2$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

vertex $(3, 2)$

max or min @ $y = 2$

axis of symmetry $x = 3$

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \leq 2, y \in \mathbb{R}\}$

$$f(x) = -2x^2 + 8x - 5$$

$$y = -2(x^2 - 4x) - 5$$

$$y = -2(x^2 - 4x + 4 - 4) - 5$$

$$y = -2(x-2)^2 + 8 - 5$$

$$= -2(x-2)^2 + 3$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

vertex $(2, 3)$

You try this one:

$$f(x) = 3x^2 + 9x - 2$$

hint: keep the fractions. Fractions are your friend.

- → complete the square
- → state the vertex
- → max or min @where.
- → axis of symmetry
- → domain
- → range

HW: pg192, Q:2,5,6,7,12ace (coefficient of one)
pg192, Q:3,4,6bcd,7bc,8bc (not a coefficient of one)