ENERGY LESSON 5

The Work-Energy Relationship

Analysis of Situations in Which Mechanical Energy is Conserved

The quantitative relationship between work and the two forms of mechanical energy is expressed by the following equation:

$$KE_i + PE_i + W_{ext} = KE_f + PE_f$$

However, there are situations in which the only forces doing work are **internal forces.**

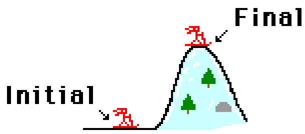
In such situations, the total mechanical energy of the object is not changed. The external work term cancels from the above equation and mechanical energy is conserved.

The previous equation is simplified to the following form:

$$KE_i + PE_i = KE_f + PE_f$$

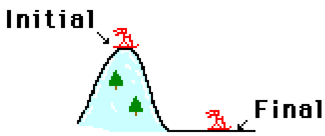
In these situations, the sum of the kinetic and potential energy is everywhere the same.

> As the potential energy is increased due to the stretch/compression of a spring or an increase in its height above the earth, the kinetic energy is decreased due to the object slowing down.
>
> As the potential energy is decreased due to the return of a spring to its rest position or a decrease in height above the earth, the kinetic energy is increased due to the object speeding up.
>
> We would say that energy is *transformed* or changes its form from kinetic energy to potential energy (or vice versa); yet the total amount present is conserved - i.e., always the same.



As the skier skis up a hill kinetic energy is lost and potential energy is gained.

$$KE_i + PE_i = KE_f + PE_f$$



As the skier skis down a hill potential energy is lost and kinetic energy is gained.

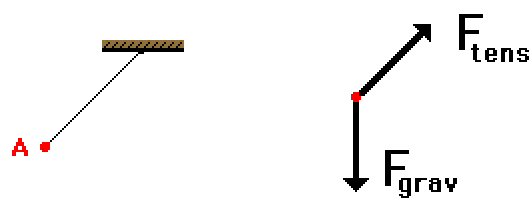
$$KE_i + PE_i = KE_f + PE_f$$

The Example of Pendulum Motion

A pendulum bob swinging *to and fro* on the end of a string.

There are only two forces acting upon the pendulum bob:

> Gravity (an internal force) acts downward and
>
> the tensional force (an external force) pulls upwards towards the pivot point.
>
> (Assume air resistance is zero)



When at location A, the pendulum bob experiences two forces. Tension is an external force, yet does no work since it acts perpendicularly to the motion.

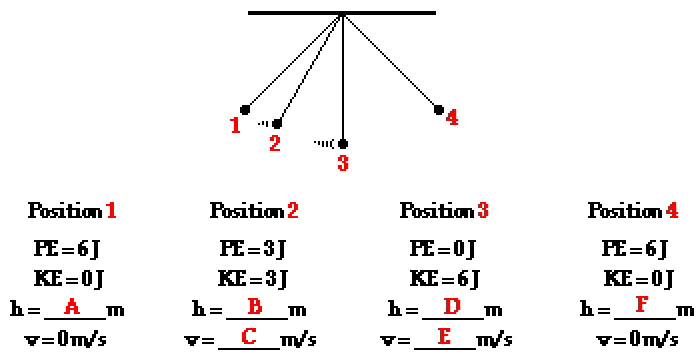
The external force does not do work since at all times it is directed at a 90-degree angle to the motion.

As the pendulum bob swings *to and fro*, its height above the table top (and in turn its speed) is constantly changing.

> As the height decreases, potential energy is lost; and simultaneously the kinetic energy is gained.
>
> Yet at all times, the sum of the potential and kinetic energies of the bob remains constant. The total mechanical energy is 6 J. There is no loss or gain of

mechanical energy; only a transformation from kinetic energy to potential energy (and vice versa).

This is depicted in the diagram below.



As the 2.0-kg pendulum bob in the above diagram swings to and fro, its height and speed change. Use energy equations and the above data to determine the blanks in the above diagram.

A: **h = 0.306 m**

PE = mgh

(6 J = 2 kg *9.8 m/s/s * h)

B: **h = 0.153 m** (3 J = 2 kg *9.8 m/s/s * h)

C: **v = 1.73 m/s**

KE = ½ mv^2

(3 J = 0.5 * 2 kg * v^2)

D: **h = 0 m** (0 J = 2 kg * 9.8 m/s/s*h)

E: **v = 2.45 m/s** (6 J = 0.5 * 2 kg * v^2)

F: **h = 0.306 m** (6 J = 2 kg * 9.8 m/s/s * h)

ANIMATION – "Energy Transformation for a Pendulum"

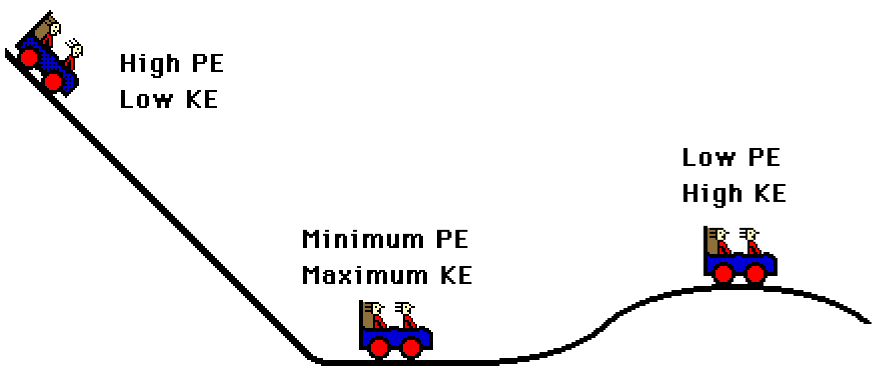
http://www.physicsclassroom.com/mmedia/

The Example of a Roller Coaster

A roller coaster operates on this same principle of energy transformation.

Once lifted to the top of the summit, the roller coaster car has a large quantity of potential energy and virtually no kinetic energy (the car is almost at rest). (Assume air resistance and friction is negligible)

> As the car descends hills and loops, its potential energy is transformed into kinetic energy as the car speeds up. As the car climbs up hills and loops, its kinetic energy is transformed into potential energy as the car slows down.
>
> Yet in the absence of external forces doing work, the total mechanical energy of the car is conserved.



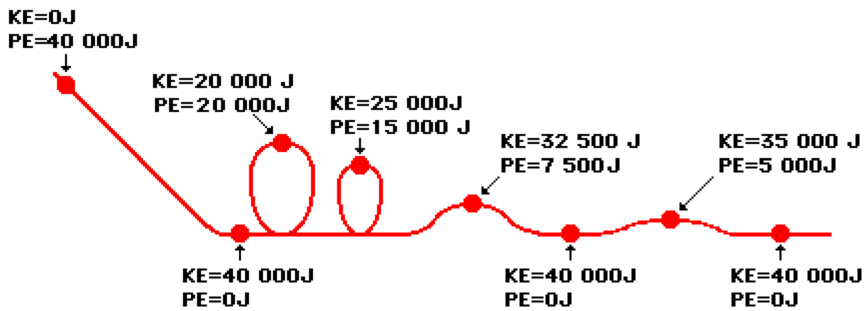
As a coaster car loses height, it gains speed; PE is transformed into KE. As a coaster car gains height it loses speed; KE is transformed into PE. The sum of the KE and PE is a constant.

Conservation of energy on a roller coaster ride means that the total amount of mechanical energy is the same at every location along the track.

The amount of kinetic energy and the amount of potential energy is constantly changing but the sum is the same everywhere.

This is illustrated in the diagram below.

The total mechanical energy of the roller coaster car is a constant value of 40 000 Joules.



ANIMATION – “Energy Transformation on a Roller Coaster"

http://www.physicsclassroom.com/mmedia/

In real life, the roller coaster car experiences the force of friction and the force of air resistance during the course of its motion.

> Friction and air resistance are both external forces and would do negative work upon the moving object causing the total mechanical energy to decrease during the course of motion.
>
> While the assumption that mechanical energy is conserved is an invalid assumption, it is a useful approximation which assists in the analysis of an otherwise complex motion.

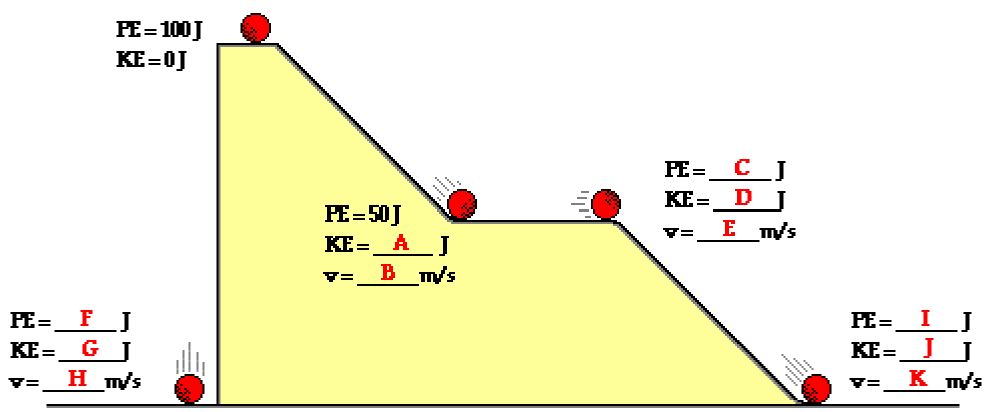
SUMMARY:

There is a relationship between work and mechanical energy change. Whenever work is done upon an object by an **external force**, there will be a change in the total mechanical energy of the object.

If only **internal forces** are doing work, there is no change in total mechanical energy; the total mechanical energy is said to be *"conserved*."

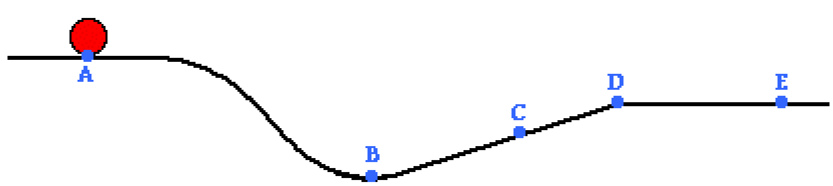
ENERGY LESSON 5 HOMEWORK

1. Consider the falling and rolling motion of the ball in the following two resistance-free situations. In one situation, the ball falls off the top of the platform to the floor. In the other situation, the ball rolls from the top of the platform along the staircase-like pathway to the floor. For each situation, indicate what type of forces are doing work upon the ball. Indicate whether the energy of the ball is conserved and explain why. Finally, fill in the blanks for the 2-kg ball.



2. If frictional forces and air resistance were acting upon the falling ball in #1 would the kinetic energy of the ball just prior to striking the ground be more, less, or equal to the value predicted in #1?

Use the following diagram to answer questions #3 - #5. Neglect the affect of resistance forces.



3. As the object moves from point A to point D across the surface, the sum of its gravitational potential and kinetic energies ____.

a. decreases, only b. decreases and then increases

c. increases and then decreases d. remains the same

4. The object will have a minimum gravitational potential energy at point ____.

a. A b. B c. C d. D e. E

5. The object's kinetic energy at point C is less than its kinetic energy at point ____.

a. A only b. A, D, and E c. B only d. D and E

6. Many drivers education books provide tables which relate a car's braking distance to the speed of the car (see table below). Utilize what you have learned about the stopping distance-velocity relationship to complete the table.

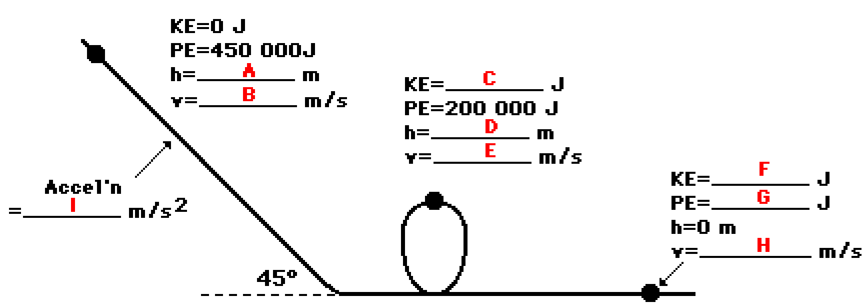
Car Speed (mph)	Braking Distance (ft)
60	240
40	a
30	b
20	27
10	c

7. Some driver's license exams have the following question.

> A car moving 50.0 km/hr skids 15.0 meters with locked brakes. How far will the car skid with locked brakes if it is moving at 150.0 km/hr?

8. Two baseballs are fired into a pile of hay. If one has twice the speed of the other, how much farther does the faster baseball penetrate? (Assume that the force of the haystack on the baseballs is constant).

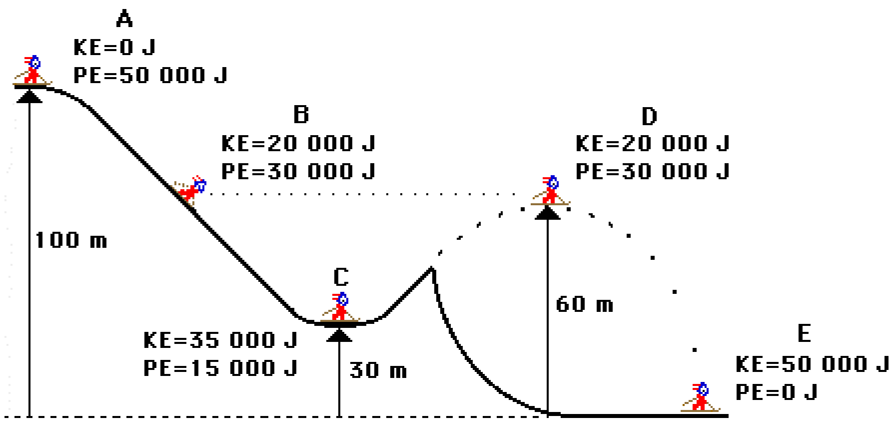
9. Use the law of conservation of energy (assume no friction) to fill in the blanks at the various marked positions for a 1000-kg roller coaster car.



(Assume 2 sig figs for answers)

10. If the angle of the initial drop in the roller coaster diagram above were 60 degrees (and all other factors were kept constant), would the speed at the bottom of the hill be any different? Explain.

11. Determine Li Ping Phar's (a mass of approximately 50 kg) speed at locations B, C, D and E.



(Assume 2 sig figs for answers)

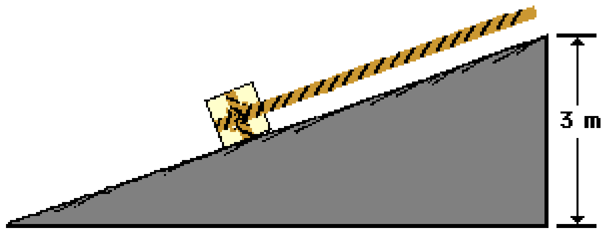
12. An object which weighs 10 N is dropped from rest from a height of 4 meters above the ground. When it has free-fallen 1 meter its total mechanical energy with respect to the ground is _____.

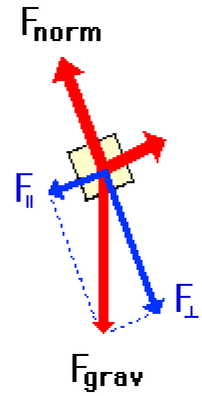
a. 2.5 J b. 10 J c. 30 J d. 40 J

13. During a certain time interval, a 20-N object free-falls 10 meters. The object gains ______ Joules of kinetic energy during this interval.

a. 10 b. 20 c. 200 d. 2000

14. A rope is attached to a 50.0-kg crate to pull it up a frictionless incline at constant speed to a height of 3.00-meters. A diagram of the situation and a free-body diagram is shown below. Note that the force of gravity has two components (parallel and perpendicular component); the parallel component balances the applied force and the perpendicular component balances the normal force.





(a) Of the forces acting upon the crate, which one(s) do work upon it?

(b) Based upon the types of forces acting upon the system and their classification as internal or external forces, is energy conserved? Explain.

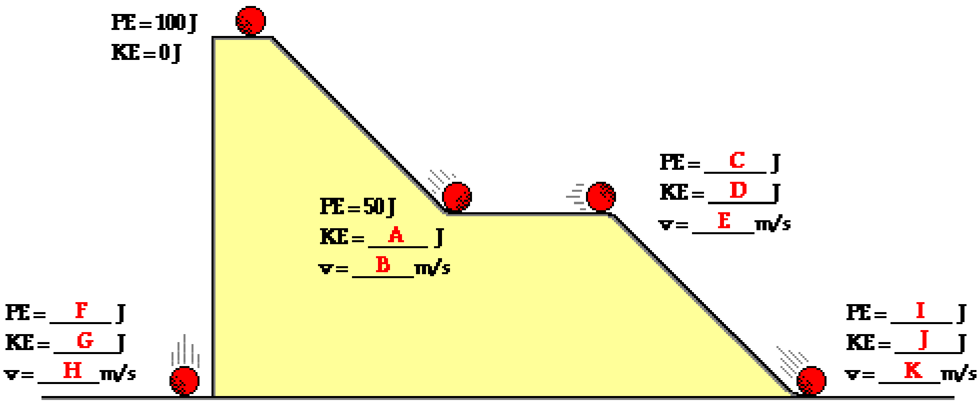
(c) Calculate the amount of work done upon the crate.

HOMEWORK KEY

A. 50 J
B. 7 m/s
C. 50 J
D. 50 J
E. 7 m/s
F. 0 J
G. 100 J
H. 10 m/s
I. 0 J
J. 100 J
K. 10 m/s
2.
3. d
4. b
5. c
6. 110 ft, 6.0 x 10^1 ft, 6.8 ft
7. 135 m
8. 4x as far
9.
a. 46m
b. 0 m/s
c. 250000 J
d. 2.0 x 10^1 m
e. 22 m/s
f. 450 000 J
g. 0 J
h. 32 m/s
i. 7.1 m/s^2
10.
11. 28m/s, 37m/s, 28m/s, 45 m/s
12. d
13. c
14. b. no, c. 1470 J

ENERGY LESSON 5 HOMEWORK

1. Consider the falling and rolling motion of the ball in the following two resistance-free situations. In one situation, the ball falls off the top of the platform to the floor. In the other situation, the ball rolls from the top of the platform along the staircase-like pathway to the floor. For each situation, indicate what type of forces are doing work upon the ball. Indicate whether the energy of the ball is conserved and explain why. Finally, fill in the blanks for the 2-kg ball.



Answer: The only force doing work is gravity. Since it is an internal or conservative force, the total mechanical energy is conserved. Thus, the 100 J of original mechanical energy is present at each position. So the KE for A is **50 J**.

The PE at the same stairstep is **50 J (C)** and thus the KE is also **50 J (D)**.

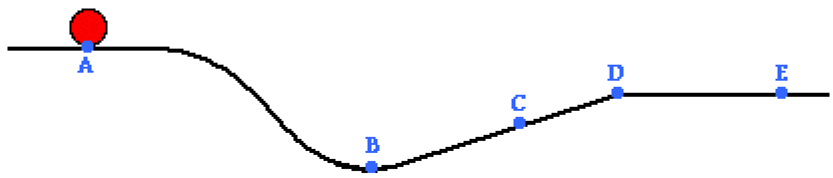
The PE at zero height is **0 J (F and I)**. And so the kinetic energy at the bottom of the hill is **100 J (G and J)**.

Using the equation **KE = 0.5*m*v^2**, the velocity can be determined to be **7 m/s for B and E** and **10 m/s for H and K**.

2. If frictional forces and air resistance were acting upon the falling ball in #1 would the kinetic energy of the ball just prior to striking the ground be more, less, or equal to the value predicted in #1?

Answer: The kinetic energy would be less in a situation that involves friction. Friction would do negative work and thus remove mechanical energy from the falling ball.

Use the following diagram to answer questions #3 - #5. Neglect the affect of resistance forces.



3. As the object moves from point A to point D across the surface, the sum of its gravitational potential and kinetic energies _____.

a. decreases, only b. decreases and then increases
c. increases and then decreases d. remains the same

Answer: The answer is **D**. The total mechanical energy (i.e., the sum of the kinetic and potential energies) is everywhere the same whenever there are no external or nonconservative forces (such as friction or air resistance) doing work.

4. The object will have a minimum gravitational potential energy at point _____.

a. A b. B c. C d. D e. E

Answer: The answer is **B**. Gravitational potential energy depends upon height (PE=m*g*h). The PE is a minimum when the height is a minimum. Position B is the lowest position in the diagram.

5. The object's kinetic energy at point C is less than its kinetic energy at point _____.

a. A only b. A, D, and E c. B only d. D and E

Answer: The answer is **C**. Since the total mechanical energy is conserved, kinetic energy (and thus, speed) will be greatest when the potential energy is smallest. Point B is the only point that is lower than point C. The reasoning would follow that point B is the point with the smallest PE, the greatest KE, and the greatest speed. Therefore, the object will have less kinetic energy at point C than at point B (only).

6. Many drivers education books provide tables which relate a car's braking distance to the speed of the car (see table below). Utilize what you have learned about the stopping distance-velocity relationship to complete the table.

Car Speed (mph)	Braking Distance (ft)
60	240
40	a
30	b
20	27
10	c

Answer: **A: 110 ft**

Compare 20 mph to 40 mph - a two-fold increase in speed. A two-fold increase in speed means a four-fold increase in stopping distance. Multiply 27 by 4.

B: 6.0×10^{1} ft

Compare 60 mph to 30 mph - a two-fold decrease in speed. A two-fold decrease in speed means a four-fold decrease in stopping distance. Divide 240 by 4.

C: 6.8 ft

Compare 20 mph to 10 mph - a two-fold decrease in speed. A two-fold decrease in speed means a four-fold decrease in stopping distance. Divide 27 by 4.

7. Some driver's license exams have the following question.

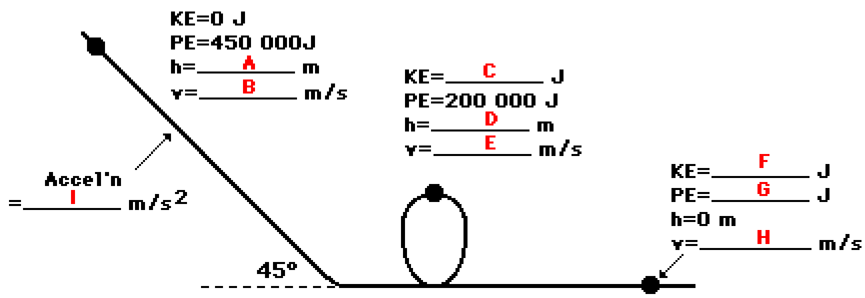
A car moving 50.0 km/hr skids 15.0 meters with locked brakes. How far will the car skid with locked brakes if it is moving at 150.0 km/hr?

Answer: The car skids **135 m**. There is a three-fold increase in the speed of the car (150 / 50 = 3). Thus, there must be a nine-fold increase in the stopping distance. Multiply 15 meters by 9.

8. Two baseballs are fired into a pile of hay. If one has twice the speed of the other, how much farther does the faster baseball penetrate? (Assume that the force of the haystack on the baseballs is constant).

Answer: The faster baseball penetrates **four times as far**. When there is a two-fold increase in speed, there is a four-fold increase in stopping distance. For constant resistance forces, stopping distance is proportional to the square of the speed.

9. Use the law of conservation of energy (assume no friction) to fill in the blanks at the various marked positions for a 1000-kg roller coaster car.



Answer: A: **h = 46 m** (from 450 000 =1000*9.8*h)

B: **v = 0 m/s** (since KE = 0 J)

C: **KE = 250 000 J** (KE + PE must equal 450 000 J)

D: **h = 2.0 x 10^1 m** (From 200 000 =1000*9.8*h)

E: **v = 22 m/s** (from 250 000 = 0.5*1000*v^2)

F: **KE = 450 000 J** (KE + PE must equal 450 000 J)

G: **PE = 0 J** (since the height is 0 m)

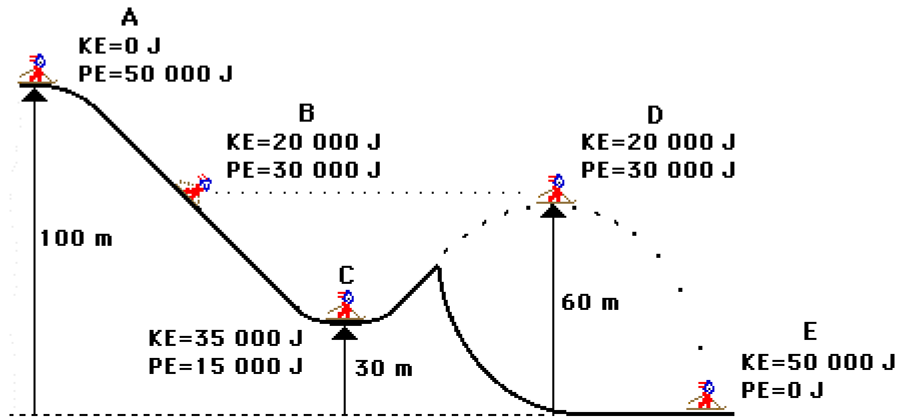
H: **v = 32 m/s** (from 500 000 = 0.5*1000*v^2)

I: **a = 7.1 m/s/s** (since a = g*sin angle)

10. If the angle of the initial drop in the roller coaster diagram above were 60 degrees (and all other factors were kept constant), would the speed at the bottom of the hill be any different? Explain.

Answer: The angle does not affect the speed at the bottom of the incline. The speed at the bottom of the incline is dependent upon the initial height of the incline. Many students believe that a smaller angle means a smaller speed at the bottom. But such students are confusing speed with acceleration. A smaller angle will lead to a smaller acceleration along the incline.

11. Determine Li Ping Phar's (a mass of approximately 50 kg) speed at locations B, C, D and E.



(Assume 2 sig figs

Answer: B: KE = 0.5 •m • v^2

20 000 J = 0.5 • (50 kg) • v^2

v = 28 m/s

C: KE = (0.5 •m • v^2

35 000 J = 0.5 • (50 kg) • v^2

v = 37 m/s

D: same as postition B

v = 28 m/s

E: KE = 0.5 •m • v^2

50 000 J = 0.5 • (50 kg) • v^2

v = 45 m/s

12. An object which weighs 10 N is dropped from rest from a height of 4 meters above the ground. When it has free-fallen 1 meter its total mechanical energy with respect to the ground is ____.

a. 2.5 J b. 10 J c. 30 J d. 40 J

Answer: The answer is **D**.

Energy is conserved in free-fall situations (no external forces doing work). Thus, the total mechanical energy initially is everywhere the same. Whatever total mechanical energy (TME) it has initially, it will maintain throughout the course of its motion. The object begins with 39.2 J of potential energy (PE = m * g * h = 1 kg * 9.8 m/s/s * 4 m = 39.2 J) and no kinetic energy. The total mechanical energy (KE + PE) is 39.2 J.

Observe that a confusion of mass (1 kg) and weight (9.8 N) will inevitably lead to the wrong answer.

13. During a certain time interval, a 20-N object free-falls 10 meters. The object gains _____ Joules of kinetic energy during this interval.

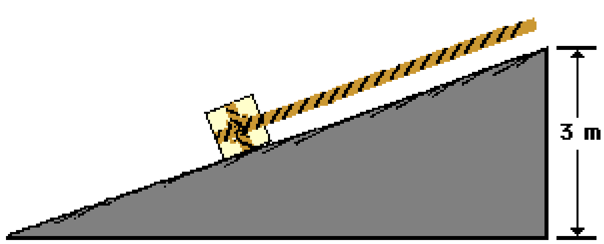
a. 10 b. 20 c. 200 d. 2000

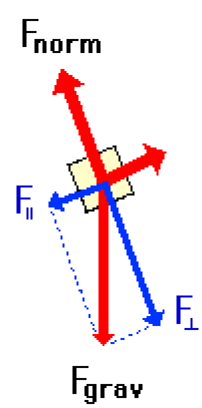
Answer: The answer is **C**.

The total amount of mechanical energy is conserved in free-fall situations (no external forces doing work). Thus, the potential energy that is lost is transformed into kinetic energy. The object loses 196 J of potential energy (PE loss = m * g * h = 20 kg * 9.8 m/s/s * 1 m = 196 J).

Observe that a confusion of mass (20 kg) and weight (196 N) will inevitably lead to the wrong answer.

14. A rope is attached to a 50.0-kg crate to pull it up a frictionless incline at constant speed to a height of 3.00-meters. A diagram of the situation and a free-body diagram is shown below. Note that the force of gravity has two components (parallel and perpendicular component); the parallel component balances the applied force and the perpendicular component balances the normal force.





(a) Of the forces acting upon the crate, which one(s) do work upon it?

(b) Based upon the types of forces acting upon the system and their classification as internal or external forces, is energy conserved? Explain.

(c) Calculate the amount of work done upon the crate.

Answer: (a) Both gravity and applied forces do work. The normal force does **not** do work since the angle between F_{norm} and the displacement is 90 degrees. (If necessary, review the lesson on work.)

(b) **No!**

The applied force is an external or nonconservative force. And since it does work, the total mechanical energy is not conserved.

(c) **W_{ext} = 1470 J**

Start with $TME_i + W_{ext} = TME_f$

$KE_i + PE_i + W_{ext} = KE_f + PE_f$

KE_i + 0 J + W_{ext} = KE_f + (50 kg) * (9.8 m/s/s) * (3 m)

($KE_i = KE_f$ since speed is constant. Thus, both KE terms can be eliminated from the equation.)

W_{ext} = (50 kg) * (9.8 m/s/s) * (3 m) = 1470 J