

Gravitational Ep and infinity

We need a new place to define Ep, that place is infinity <= infinitely far from all masses.

$$E_p = -\frac{Gm_1m_2}{r}$$

New formula for Ep based on Ep = 0 J at infinity

Calculate the Ep of a 5000 kg cat at a distance of 3.0×10^7 m from Earth's centre.

$$E_p = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11} (5000)(5.98 \times 10^{24})}{3.0 \times 10^7}$$

$$= -6.65 \times 10^{10} \text{ J}$$

Work done:

USE THE WORK ENERGY THEOREM!!

$$W = \Delta E = E_{pf} - E_{pi} \quad \left| \quad = \frac{Gm_1m_2}{r_f} - \left(-\frac{Gm_1m_2}{r_i} \right) = \frac{Gm_1m_2}{r_i} - \frac{Gm_1m_2}{r_f}$$

A mass of 5000 kg is moved from 2.0×10^7 m distance to 3.0×10^7 m distance (all distances are from centre of Earth), find the work done.

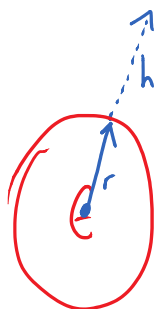
$$m = 5000 \quad 2 \times 10^7 \rightarrow 3 \times 10^7 \quad \hookrightarrow W = ?$$

$$W = \Delta E = E_{pf} - E_{pi} = -\frac{6.67 \times 10^{-11} (5000)(5.98 \times 10^{24})}{3 \times 10^7} + \frac{6.67 \times 10^{-11} (5000)(5.98 \times 10^{24})}{2 \times 10^7}$$

$$= 3.32 \times 10^{10} \text{ J}$$

A 10 kg cat on the surface of the Earth has 4.0×10^8 J of work done on it, to what maximum height will it rise?

A 10 kg cat on the surface of the Earth has $4.0 \times 10^8 \text{ J}$ of work done on it, to what maximum height will it rise?



$$W = \Delta E = Gm_1m_2 \left(\frac{1}{r_0} - \frac{1}{r_f} \right)$$

X^{-1}

$$4 \times 10^8 = 6.67 \times 10^{-11} (10) (5.98 \times 10^{24}) \left(\frac{1}{6.38 \times 10^6} - \frac{1}{r_f} \right)$$

$$\frac{4 \times 10^8}{6.67 \times 10^{-11} (10) (5.98 \times 10^{24})} - \frac{1}{6.38 \times 10^6} = -\frac{1}{r_f}$$

$$r_f = 17.7 \text{ Mm.}$$

$$\begin{aligned} r_f &= 17.7 \text{ Mm} \\ \text{height} &= r_f - r_e \\ &= 17.7 \times 10^6 - 6.38 \times 10^6 \\ &= 11.3 \text{ Mm.} \end{aligned}$$

The Law of Conservation of Energy still applies:

$$E_{p0} + E_{k0} = E_{pf} + E_{kf} + Q \text{ but } Q = 0 \text{ J}$$

↑ ↑

Typical situations involve:

- An object moving in space crashing into the Earth (any planet)
- An object moving in space to some closer distance to the Earth (any planet)
- An object on the Earth moving into space

comet/meteor/asteroid

A comet of mass $1.0 \times 10^7 \text{ kg}$ is $4.0 \times 10^9 \text{ m}$ from Earth's centre, it is moving at 2500 m/s and crashes into the Earth's surface, what is the impact speed?

Conservation of Energy.

↳ V_f

$$E_{p0} + E_{k0} = E_{pf} + E_{kf}$$

$$-\frac{Gm_1m_2}{r_0} + \frac{mv_0^2}{2} = -\frac{Gm_1m_2}{r_f} + \frac{mV_f^2}{2}$$

$$\frac{-6.67 \times 10^{-11} (5.98 \times 10^{24})}{4 \times 10^9} + \frac{2500^2}{2} = \frac{-6.67 \times 10^{-11} (5.98 \times 10^{24})}{6.38 \times 10^6} + \frac{V_f^2}{2}$$

$$V_f = 11.4 \frac{\text{km}}{\text{s}}$$

A cat is blasted off of the moon at 1.3 km/s from the surface, to what height
 Will it rise before coming to rest?

$$h = ? \quad E_{p0} + E_{k0} = E_{pf} + E_{kf}$$

$$\frac{-6.67 \times 10^{-11} (7.35 \times 10^{22})}{1.74 \times 10^6} + \frac{(1.3 \times 10^3)^2}{2} = \frac{-6.67 \times 10^{-11} (7.35 \times 10^{22})}{r}$$

$$r = 2.49 \text{ Mm}$$

$$h = 2.49 \times 10^6 - 1.74 \times 10^6$$

$$= 750 \text{ km.}$$

Escape Velocity: is defined as the velocity at a planet's surface necessary
 To ESCAPE to infinity, you can stop when you reach infinity. Escape Velocity
 Is found using Conservation of Energy

$$E_{p0} + E_{k0} = E_{pf} + E_{kf}$$

$$\frac{-Gm_1 m_2}{r_0} + \frac{1}{2} m v^2 = 0 + 0$$

$$\frac{v^2}{2} = \frac{Gm_1}{r_0}$$

$$v = \sqrt{\frac{2Gm}{r_0}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^6}}$$

$$= 11.2 \text{ km/s}$$

In ORBIT there is a relationship between

E_p and E_k .

$$F_c = F_g$$

$$E_k = \frac{mv^2}{2} = \frac{m}{2} \left[\frac{Gm}{r} \right]$$

$$F_c = F_g$$
$$m\left(\frac{v^2}{r}\right) = \frac{Gm_1m_2}{r^2}$$

$$v^2 = \frac{Gm}{r}$$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Gm}{r}\right)$$

$$= \frac{Gm^2}{2r}$$

$$E_p = -\frac{Gm^2}{r}$$

$$\therefore E_k = -\frac{1}{2}E_p \quad \square$$

mind = blown