Chapter 6 Rational Expressions and Equations

6.1 Rational Expressions

KEY IDEAS

- A rational number is a ratio of integers, $\frac{a}{b}$, $b \neq 0$. For example, $\frac{22}{7}$, $\frac{-2}{3}$, $\frac{15}{10}$, 7, 0.
- A rational expression is a ratio of polynomials, $\frac{p}{q}$, $q \neq 0$.

  For example, $\frac{x - 3}{x^2 + 4}$, $\frac{x^2 + 7x + 6}{x^2 - 36}$, $\frac{2}{4x + 8}$, $\frac{3xy}{x - y^2}$.
- The value for a variable in a rational expression can be any real value except for non-permissible values. Non-permissible values are values of the variable(s) that make the denominator of a rational expression equal to zero.

  For example, in the rational expression $\frac{x^2 + 7x + 6}{x^2 - 36}$, $x \neq \pm 6$. The non-permissible values are 6 and −6, since $x^2 - 36 = (x + 6)(x - 6)$, and dividing by zero is undefined.
- Like fractions, rational expressions can be simplified to lowest terms. To simplify a rational expression,
  - fully factor the numerator and denominator
  - determine any non-permissible values
  - divide the numerator and the denominator by all identical factors
- Use these properties when working with rational expressions:
  - Any number divided by itself is 1.
    For example, $\frac{14}{14} = 1$.
  - Any polynomial divided by itself is 1 (except for the non-permissible values of the variable).
    For example, $\frac{x^2 - 2x + 1}{x^2 - 2x + 1} = 1$, $x \neq 1$
  - Any number divided by its opposite is −1.
    For example, $\frac{29}{-29} = -1$.
  - Any polynomial divided by its opposite is also −1 (except for the non-permissible values of the variable).
    The opposite of polynomial $a - b$ is $b - a$ because $-1(a - b) = -a + b$, or $b - a$.
    For example, $\frac{x - 5}{5 - x} = -1$, $x \neq 5$.

Working Example 1: Determine Non-Permissible Values

For each rational expression, determine all non-permissible values.

a) $\frac{1}{2x - 3}$ **b)** $\frac{1}{x^2 - 5x + 6}$ **c)** $\frac{x^2 - 9x + 14}{x^2 - 25}$

Solution

Non-permissible values occur because dividing by zero is not allowed (undefined). To find the non-permissible values, set the denominator equal to zero and solve for the variable(s).

a) In the rational expression $\frac{1}{2x - 3}$, the denominator is ________________.

$2x - 3 = 0$

$2x =$ ________

$x =$ ________

Therefore, the non-permissible value of x is ________. Write this as $x \neq$ ________.

b) In the rational expression $\frac{1}{x^2 - 5x + 6}$, the denominator is ________________.

Determine the roots, or zeros, of the quadratic expression in the denominator to determine the non-permissible values.

$x^2 - 5x + 6 = 0$

(________________)(________________) = 0

________________ = 0 or ________________ = 0

$x =$ ________ $x =$ ________

Therefore, the non-permissible values of x are ________ and ________.

Write this as $x \neq$ ________, ________.

c) In the rational expression $\frac{x^2 - 9x + 14}{x^2 - 25}$, the denominator is ________________.

Ignore the numerator when finding non-permissible values.

$x^2 - 25 = 0$

(________________)(________________) = 0

________________ = 0 or ________________ = 0

$x =$ ________ $x =$ ________

Therefore, the non-permissible values of x are ________ and ________.

Write this as $x \neq \pm$ ________.

Working Example 2: Simplify a Rational Expression

Simplify each rational expression. State the non-permissible values.

a) $\dfrac{\frac{4}{3}\pi r^3}{4\pi r^2}$ **b)** $\dfrac{x^2 - 1}{x^2 + 3x + 2}$ **c)** $\dfrac{2x^3 - 4x^2 - 30x}{4x^2 - 20}$

Solution

a) $\dfrac{\frac{4}{3}\pi r^3}{4\pi r^2}$

π is a constant, not a variable.

Determine the non-permissible values.

$r =$ ________

Express the rational expression in simplest form.

Look for common factors in the numerator and the denominator.

$$\frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\cancel{4\pi r^2}\left(\frac{1}{3}\right)r}{\cancel{4\pi r^2}}$$

$=$ ________, $r \neq 0$

b) $\dfrac{x^2 - 1}{x^2 + 3x + 2}$

Factor the numerator and denominator.

$$\frac{x^2 - 1}{x^2 + 3x + 2} = \frac{(\square)(\square)}{(x + 1)(x + 2)}$$

Determine the non-permissible values.

________ $= 0$ or ________ $= 0$

$x =$ ________ $x =$ ________

The non-permissible values are $x =$ ________, ________.

Simplify the expression.

$$\frac{\cancel{(x + 1)}(x - 1)}{\cancel{(x + 1)}(x + 2)}$$

$= \dfrac{(\square)}{(\square)}$, $x \neq$ ________, ________

c) $\dfrac{2x^3 - 4x^2 - 30x}{4x^2 - 20x}$

Factor the numerator and denominator.

Check for common factors first!

$\dfrac{2x^3 - 4x^2 - 30x}{4x^2 - 20x} = \dfrac{\boxed{}}{4x(x-5)}$

Determine the non-permissible values.

________ $= 0$ or ________ $= 0$

$x =$ ________ $x =$ ________

The non-permissible values are $x =$ ________, ________.

Simplify the expression.

$\dfrac{\cancel{2x}\cancel{(x-5)}(x+3)}{{}_{2}\cancel{4x}\cancel{(x-5)}}$

$= \dfrac{(\boxed{})}{2}$, $x \neq$ ________, ________

Working Example 3: Rational Expressions With Pairs of Non-Permissible Values

For the rational expression $\dfrac{(w+1)(h-8)}{w-4h}$, determine all non-permissible values.

Solution

To find the non-permissible values, set the denominator equal to zero and solve for one variable.

$w - 4h = 0$

$w =$ ________

The value of one variable depends on the value of the other variable. Non-permissible values will come in pairs. For example,

When $h = 1$, $w = 4$. When $h = 5$, $w =$ ________.

When $h = 8.25$, $w =$ ________.

The non-permissible values of w are $4h$. Write this as $w \neq$ ________.

Working Example 4: Recognize Additive Inverses (Opposites)

Simplify each rational expression. State the non-permissible values.

a) $\frac{x^2 - 81}{9 - x}$ **b)** $\frac{12 - 3x}{x^2 + x - 20}$

Solution

a) Factor the numerator and denominator.

$$\frac{x^2 - 81}{9 - x} = \frac{(\boxed{})(\boxed{})}{9 - x}$$

Look for identical factors to help simplify the answer.

Determine the non-permissible values of x.

$9 - x = 0$

$x =$ ________

The non-permissible value is $x =$ ________.

To simplify, cancel factors common to the numerator and the denominator.

$$\frac{(x - 9)(x + 9)}{9 - x}$$

Simplify.

$$\frac{\cancel{(x - 9)}(x + 9)}{-1\cancel{(x - 9)}} = \frac{(x + 9)}{-1}$$

$$= -(x + 9),\ x \neq 9$$

$x - 9$ and $9 - x$ are opposites.
$(9 - x) = -1(-9 + x)$
$= -1(x - 9)$

b) Factor the numerator and denominator. Look for common factors first.

$$\frac{12 - 3x}{x^2 + x - 20} = \frac{\boxed{}}{(x + 5)(x - 4)}$$

Determine the non-permissible values of x.

________ $= 0$ or ________ $= 0$

$x =$ ________ $x =$ ________

The non-permissible values are $x =$ ________, ________.

Simplify to express the fraction in lowest terms.

$$\frac{3(4 - x)}{(x + 5)(x - 4)}$$

Look for identical factors and opposite factors.

In the expression $\frac{3(4 - x)}{(x + 5)(x - 4)}$, the pair of opposites is ____________ and ____________.

Cancel the opposites to simplify.

$$\frac{3\cancel{(4 - x)}}{(x + 5)(-1)\cancel{(x - 4)}} = \frac{3}{-(x + 5)},\ x \neq -5, 4$$

Why did the factor (–1) appear in the denominator?

This method is a shortcut that relies on factoring out –1 from one of the pair of opposites. See pages 314–315 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. For each rational expression, determine all non-permissible values of the variable(s).

a) $\frac{1}{x + 4}$

The denominator is ________.

$x \neq$ ________

b) $\frac{1}{6x - 3}$

The denominator is ________.

$x \neq$ ________

c) $\frac{1}{x(x - 1)}$

$x \neq$ ________, ________

d) $\frac{1}{(x + 2)(3x - 2)}$

$x \neq$ ________, ________

e) $\frac{1}{x^2 + 3x}$

Factor the denominator. Then, determine the non-permissible values.

$x \neq$ ________, ________

f) $\frac{1}{x^2 + 4x - 21}$

$x \neq$ ________, ________

g) $\frac{6x + 3}{5x - 10x^2}$

The numerator is not needed when finding non-permissible values.

$x \neq$ ________, ________

h) $\frac{x - 200}{x^2 - 100}$

$x \neq$ ________, ________

2. Simplify each rational expression. State any non-permissible values for the variables.

a) $\dfrac{4x(x + 1)}{2(x + 1)}$

b) $\dfrac{6x^2(2x - 3)}{2x(2x - 3)}$

Determine the non-permissible values before simplifying.

c) $\dfrac{(x - 7)(x + 4)}{(x + 4)(x + 7)}$

d) $\dfrac{x(2x + 7)(x + 6)}{x^2(x + 6)}$

3. Simplify each rational expression. State any non-permissible values for the variables.

a) $\dfrac{8x^2y^3}{12y^2}$

b) $\dfrac{\pi r^2}{2\pi rh}$

c) $\dfrac{10x^2 + 70x}{5x^2 + 5x}$

Factor fully. Remember to look for common factors first.

d) $\dfrac{a^2b - ab^3}{5ab + 25a^2b}$

Also try #6 and 8 on page 318 of *Pre-Calculus 11*.

4. Simplify and state the non-permissible values for the variables.

a) $\dfrac{x^2 - 25}{x^2 + 5x - 50}$

Factor first. Then, determine non-permissible values before simplifying.

b) $\dfrac{x^2 + 5x - 6}{x^2 + 6x}$

c) $\dfrac{2x^2 - 4x - 70}{4x - 28}$

Look for common factors first.

d) $\dfrac{6x^2 + 18x + 12}{3x + 3}$

e) $\dfrac{a^2 - a - 2}{a^2 + 5a - 14}$

f) $\dfrac{z^2 + 9z + 18}{z^2 - 3z - 18}$

g) $\dfrac{2x^2 - 5x - 6}{x^2 - 5x - 6}$

h) $\dfrac{3x^2 - 8x - 3}{3x^2 - 15x + 18}$

5. State the opposite of each binomial.
What is the result when a binomial is divided by its opposite?

a) $x - 1$

The opposite of $x - 1$ is ______________. So, $\dfrac{(x - 1)}{(\boxed{})}$ = ________, $x \neq 1$.

b) $2 - x$

c) $2a - b$

d) $-5x - 2$

6. Simplify each rational expression. State any non-permissible values for the variables.

a) $\dfrac{3a(a - 7)}{(7 + a)(7 - a)}$

b) $\dfrac{x^2 - 81}{18x - 2x^2}$

c) $\dfrac{-12x + 4x^2}{x^2 - 6x + 9}$

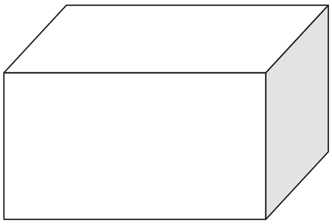
d) $\dfrac{a^2 - 2ab}{4b^2 - a^2}$

This question will help you with #25 on page 320 of *Pre-Calculus 11*.

Apply

7. A rectangular box has length x cm. Its width is 2 cm shorter than its length. Its height is 4 cm shorter than its length.

a) Determine an expression, in lowest terms, for the ratio of the surface area to the volume of the box.



length: ________ width: ________ height: ________

Write expressions for the surface area and the volume of the box.

$SA =$ ____________________________

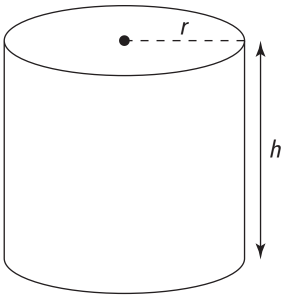
$V =$ ____________________________

$\frac{SA}{V} = \frac{\boxed{}}{\boxed{}}$

b) What are the non-permissible values of the variable x? What meaning do the non-permissible values have in the context of the problem? Are there any further restrictions on the variable that result from the context rather than the algebra?

For more application questions, see pages 318–319 of *Pre-Calculus 11*.

8. In manufacturing, you often need to know the ratio of the surface area to the volume of a container. Determine the ratio of surface area to volume of a cylinder with radius r and height h. Simplify this ratio and give any non-permissible values of the variables.



Formula for SA of a cylinder: ________________

Formula for V of a cylinder: ________________

$$\frac{SA}{V} = \frac{\boxed{}}{\boxed{}}$$

9. Consider the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a) Does the quadratic formula contain a rational expression? Why or why not?

b) What are the non-permissible values of the denominator? What meaning do the non-permissible values have in the context of solving quadratic equations?

10. a) State the non-permissible values for $\frac{1}{x^2 - 4x + 3}$.

b) Create a rational expression in which $x \neq 2, 3$.

c) Simplify and state the non-permissible values for $\frac{x^2 + 5x - 24}{x + 3}$.

d) Create a rational expression that is equivalent to $(x - 8)$ in which $x \neq -1$.

Connect

11. Is the following work correct? Explain.

$$\frac{x + 10}{10} = \frac{x + \cancel{10}}{\cancel{10}}$$
$$= x$$

6.2 Multiplying and Dividing Rational Expressions

KEY IDEAS

- A rational expression is a ratio of polynomials, $\frac{p}{q}$, $q \neq 0$.

  For example, $\frac{x + 3}{x + 7}$, $\frac{x^2 + 3x + 2}{x^2 - 4}$, $\frac{6.5}{3.6x + 9.1}$, $\frac{3x + y}{x - y}$.

- Like fractions, rational expressions can be multiplied and divided.
  – The product is the result of multiplying.
  – The quotient is the result of dividing.
  – Always express the product or quotient in simplest form.

- To multiply two rational expressions,
  – factor each numerator and denominator
  – determine any non-permissible values
  – multiply the numerators together and the denominators together
  – simplify factors (not terms) common to both the numerator and the denominator
  For example,

Correct	Incorrect
$\frac{\cancel{(x + 4)}(x - 1)}{3x\cancel{(x + 4)}} = \frac{x - 1}{3x}$	$\frac{\cancel{x} - 1}{3\cancel{x}} = \frac{-1}{3}$

- To divide one rational expression by another rational expression, multiply the first by the reciprocal of the second. Then, proceed as for multiplication.

$$\frac{A}{B} \div \frac{C}{D} = \frac{\frac{A}{B}}{\frac{C}{D}}$$
$$= \frac{A}{B} \times \frac{D}{C}$$

- The non-permissible values in a division question arise every time you divide. In the example above, there are two kinds of division. Expressions B and D are in the denominator of the original expression. A division sign (÷) tells you to divide by expression C. Therefore, B, D, and C must all be considered when determining non-permissible values for the variable(s).

  For example, in

$$\frac{A}{B} \div \frac{C}{D} = \frac{\frac{A}{B}}{\frac{C}{D}}$$

  the non-permissible values are $B \neq 0$, $D \neq 0$, and $C \neq 0$.

Working Example 1: Multiply Rational Numbers and Rational Expressions

Determine each product. Identify any non-permissible values of the variable.

a) $\frac{5}{9} \times \frac{3}{10}$

b) $\frac{x + 1}{x^2 - 5x + 6} \times \frac{x - 2}{x^2 + 5x + 4}$

Solution

a) Factor the numerator and denominator.

$$\frac{5}{9} \times \frac{3}{10} = \frac{5}{3(3)} \times \frac{2}{2(5)}$$

Write each factor as a product of prime numbers.

There are no variables, so there are no non-permissible values.
Multiply the numerators together and the denominators together.
To simplify, cancel factors common to both the numerator and the denominator.

$$= \frac{(5)(2)}{(3)(3)(2)(5)}$$

$$= \frac{\square}{\square}$$

Remember that identical factors cancel to 1.

b) Factor the numerator and denominator. Look for binomial factors.

$$\frac{x + 1}{x^2 - 5x + 6} \times \frac{x - 2}{x^2 + 5x + 4}$$

$$= \frac{(x + 1)}{(x - 3)(\square)} \times \frac{(x - 2)}{(x + 4)(\square)}$$

Identify any non-permissible values.

$x - 3 = 0$ ________ $= 0$

$x =$ ________ $x =$ ________

$x + 4 = 0$ ________ $= 0$

$x =$ ________ $x =$ ________

The non-permissible values are $x =$ ________, ________, ________, and ________.

Multiply the numerators together and the denominators together.

To simplify, cancel factors common to both the numerator and the denominator.

$$= \frac{(x + 1)(x - 2)}{(x - 3)(\square)(x + 4)(\square)}$$

$$= \frac{\square}{\square}, x \neq -4, -1, 2, 3$$

Remember that the quotient of identical factors is 1.
If all the factors in the numerator cancel, the numerator is 1.

Working Example 2: Identify Non-Permissible Values

State the non-permissible values of the variable(s).

$$\frac{ab^2}{c^2d} \div \frac{(e-2)(f+1)}{g(h-6)}$$

Solution

Identify (circle or highlight) all forms of division in the expression.

The denominators are ________________ and ________________.

Also divide by ________________ (the numerator of the second expression).

Set each of the relevant factors equal to zero to determine the non-permissible values.

$c^2 = 0$ $\qquad$ $d =$ ________

$c =$ ________

$e - 2 = 0$ $\qquad$ $f + 1 = 0$

$e =$ ________ $\qquad$ $f =$ ________

$g =$ ________ $\qquad$ $h - 6 = 0$

$h =$ ________

The non-permissible values are $c =$ ________, $d =$ ________, $e =$ ________, $f =$ ________, $g =$ ________, and $h =$ ________.

Working Example 3: Divide Rational Numbers and Rational Expressions

Determine each quotient. Identify any non-permissible values of the variable.

a) $\frac{3}{4} \div \frac{9}{2}$ $\qquad$ **b)** $\frac{x+3}{x-3} \div \frac{x}{4x-12}$ $\qquad$ **c)** $\frac{x-7}{x+4} \div \frac{x^2-2x-15}{x^2-x-20}$

Solution

a) To divide, multiply by the reciprocal.

$$\frac{3}{4} \div \frac{9}{2} = \frac{3}{4} \times \frac{2}{9}$$

$$= \frac{3}{(2)(2)} \times \frac{2}{(3)(3)}$$

$$= \frac{\square}{\square}$$

b) Factor each numerator and denominator fully.

$\frac{x+3}{x-3} \div \frac{x}{4x-12} = \frac{x+3}{x-3} \div \frac{x}{4(x-3)}$

Determine the non-permissible values.

The denominators are ________________ and ________________.

Also divide by ________________.

The non-permissible values are ________________ and ________________.

To divide, multiply by the reciprocal.

$= \frac{x+3}{x-3} \times \frac{(\boxed{})}{(\boxed{})}$

$= \frac{(x+3)(\boxed{})}{(x-3)(\boxed{})}$

$= \frac{\boxed{}}{\boxed{}}, x \neq 3, 0$

c) Factor each numerator and denominator fully. Then, determine the non-permissible values.

$\frac{x-7}{x+4} \div \frac{x^2-2x-15}{x^2-x-20} = \frac{x-7}{x+4} \div \frac{(x-5)(\boxed{})}{(x-5)(\boxed{})}$

$x + 4 = 0$ $\qquad$ $x - 5 = 0$

$x =$ ________ $\qquad$ $x =$ ________

________________ $= 0$ $\qquad$ ________________ $= 0$

$x =$ ________ $\qquad$ $x =$ ________

To divide, multiply by the reciprocal.

$= \frac{x-7}{x+4} \times \frac{(\boxed{})(\boxed{})}{(\boxed{})(\boxed{})}$

$= \frac{(x-7)}{(x+4)} \frac{(\boxed{})(\boxed{})}{(\boxed{})(\boxed{})}$

$= \frac{\boxed{}}{\boxed{}}, x \neq -4, -3, 5$

Check Your Understanding

Practise

1. What are the non-permissible value(s) for the variable(s) in each product?

a) $\frac{1}{x-2} \times \frac{3}{x-4}$

$x \neq$ ________, ________

b) $\frac{\pi r^2}{2\pi r} \times \frac{\pi rh}{r+h}$

$r \neq$ ________, ________

c) $\frac{x^2+3x+2}{x^2-1} \times \frac{1}{3x+2}$

d) $\frac{y-8}{y^2+7x+6} \times \frac{y+4}{y^2+16y+60}$

2. Write the reciprocal of each rational expression.

a) $\frac{3}{7x}$

b) $\frac{2x-7}{x+4}$

c) $\frac{x^2+2x+1}{6x-3}$

d) $49 - x^2$

3. Determine the non-permissible value(s) of the variable for each quotient.

a) $\frac{l+1}{4l} \div \frac{(w-2)^2}{6w}$

The denominators are ________________ and ________________.

Also divide by ________________ (the numerator of the second expression).

The non-permissible values are ___________________________.

b) $\frac{x-1}{x^2-4} \div \frac{2x-3}{x+5}$

The denominators are ________________ and ________________.

Also divide by ________________ (the numerator of the second expression).

The non-permissible values are ___________________________.

4. Write each product in simplest form. Determine all non-permissible values of the variable(s).

a) $\frac{3x^3}{y^2} \times \frac{y^3}{6x}$

b) $\frac{5x^2}{x-3} \times \frac{x-3}{10x}$

Always identify the non-permissible values before simplifying.

c) $\frac{x-6}{x+2} \times \frac{x+6}{x-6}$

d) $\frac{x+1}{x-2} \times \frac{x^2-x-6}{x-4}$

Put brackets around each binomial factor.

e) $\frac{2x-8}{x+3} \times \frac{x^2+4x+3}{x-4}$

f) $\frac{x^2+6x-40}{x^2-100} \times \frac{10x-x^2}{x^2+x-20}$

Also try #2 on page 327 of *Pre-Calculus 11*.

5. Write each quotient in simplest form. State any non-permissible values for the variables.

a) $\dfrac{x + 2}{4x - 5} \div \dfrac{2x + 4}{4x - 5}$

Factor the numerators and denominators.

Multiply by the reciprocal.

Determine the non-permissible values of x.

Multiply.

Simplify.

b) $\dfrac{a^2b}{c - 1} \div \dfrac{bc - b}{a}$

c) $\dfrac{x^2 + 8x + 12}{x^2 - 15x + 56} \div \dfrac{3x + 6}{x - 7}$

d) $\dfrac{x^2 - 3x - 18}{x^2 + 6x + 9} \div \dfrac{x^2 + 3x + 2}{x^2 + 8x + 15}$

Also try #4 and 8 on page 327 of *Pre-Calculus 11*.

6. Simplify and state the non-permissible values for the variable.

a) $\frac{x^2 + 16x + 64}{x^2 - 4x} \times \frac{x - 4}{x + 8}$

Factor the numerators and denominators.

Determine the non-permissible values of x.

Multiply.

Simplify.

b) $\frac{x^2 - 17x + 72}{x^2 - 4x + 3} \times \frac{9 - x^2}{x^2 - 6x - 27}$

c) $\frac{x + 11}{2x} \div \frac{121 - x^2}{x^2 + 4x}$

Factor the numerators and denominators.

Multiply by the reciprocal.

Determine the non-permissible values of x.

Multiply.

Simplify.

d) $\frac{x^2 - 5x - 50}{144 - 24x + x^2} \div \frac{5x + 25}{x^2 - 20x + 96}$

Apply

7. Simplify the following expressions. Identify any non-permissible values of the variable.

a) $\dfrac{4x + 20}{x + 6} \times \dfrac{x - 4}{x^2 - 25} \div \dfrac{x + 6}{2x - 8}$

Factor the numerators and denominators.

Multiply by the reciprocal.

Determine the non-permissible values of x.

Multiply.

Simplify.

b) $\dfrac{x^2 + 2x + 1}{2x + 1} \div \dfrac{x^2 - 3x}{x + 10} \times \dfrac{2x^2 - 5x - 3}{x^2 + x}$

c) $\dfrac{9x - x^3}{10x - 10} \div \dfrac{x^2 - 11x + 24}{3x^2 - 4x + 1} \div \dfrac{45x + 5x^2}{x - 8}$

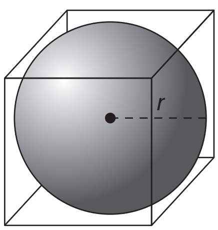
8. There is at least one error in Jaime's solution. Circle the error(s).
Complete the question correctly in the space provided.

Jaime's Solution	Correct Solution
$\dfrac{x^2 + \cancel{4x + 4}}{\cancel{4x + 4}} \times \dfrac{x + 1}{x + 2}$	$\dfrac{x^2 + 4x + 4}{4x + 4} \times \dfrac{x + 1}{x + 2}$
$= \dfrac{x^2 (x + 1)}{x + 2}$	$=$
$= \dfrac{x^3 + 1}{x + 2}, x \neq -1, -2$	

9. There is at least one error in Kelly's solution. Circle the error(s).
Complete the question correctly in the space provided.

Kelly's Solution	Correct Solution
$\dfrac{x - 4}{x + 6} \div \dfrac{2x + 6}{2x - 8}$	$\dfrac{x - 4}{x + 6} \div \dfrac{2x + 6}{2x - 8}$
$= \dfrac{\cancel{x - 4}}{x + 6} \div \dfrac{2(x + 3)}{2(\cancel{x - 4})}$	$=$
$= \dfrac{x + 3}{x + 6}, x \neq -6, 4$	

10. A sphere is contained in a rectangular box such that the sides of the sphere are touching the sides of the box. What fraction of the volume of the box does the sphere occupy? Model the situation using a quotient of rational expressions, and then solve.



$V_{box} = l \times w \times h$
$V_{sphere} = \dfrac{4\pi r^3}{3}$

See pages 328–329 of *Pre-Calculus 11* for more application questions.

Connect

11. Students are given the following expression to simplify: $\frac{2x-7}{x+6} \times \frac{x^2+3x-18}{x-5}$

Heather begins her solution by factoring the quadratic, as follows:

$$\frac{2x-7}{x+6} \times \frac{x^2+3x-18}{x-5} = \frac{2x-7}{x+6} \times \frac{(\boxed{})(\boxed{})}{x-5}$$

$$=$$

Shervin remembers that simplifying rational expressions is like simplifying rational numbers. He multiplies the numerators and the denominators first, and then looks for common factors.

For example, $\frac{10}{3} \times \frac{6}{5} = \frac{60}{15} = 4$.

He decides to try the same process with rational expressions. He begins his solution by using the distributive law to multiply the numerators and denominators together, as follows:

$$\frac{2x-7}{x+6} \times \frac{x^2+3x-18}{x-5} = \frac{2x^3-7x^2+6x^2-36x-21x+126}{x^2-5x+6x-30}$$

$$=$$

Complete Heather's and Shervin's solutions. Whose solution is more efficient? Why?

6.3 Adding and Subtracting Rational Expressions

KEY IDEAS

- To add (or subtract) rational expressions,
  - factor each denominator if necessary
  - express all rational expressions with a common denominator
  - add (or subtract) the numerators; the denominator does not change
- In mathematics, you often group like things together, but keep unlike things separate. For example, group together like terms ($2x + 5x = 7x$) but keep unlike terms separate ($2x + 5y = 2x + 5y$).
  - The same idea holds for fractions (rational numbers). Fractions with the same denominator are alike, and you can group them together.

$$\frac{5}{12} + \frac{2}{12} = \frac{5 + 2}{12} = \frac{7}{12}$$

Add (or subtract) the numerators. The denominator does not change.

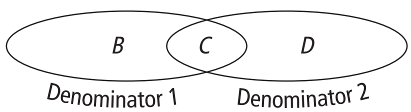
  - Similarly, rational expressions with the same denominator are alike, and you can group them together.

$$\frac{5x}{x + 12} + \frac{2x - 3}{x + 12} = \frac{5x + (2x - 3)}{x + 12} = \frac{7x - 3}{x + 12}$$

Add (or subtract) the numerators. The denominator does not change.

- If rational expressions have different denominators, rewrite them as equivalent expressions with the least common denominator (LCD):
  - Factor each denominator. Start with the factors from the first denominator, and then include any factors from the second (and subsequent) denominator that are not already represented.

$$\frac{A}{(B)(C)} + \frac{E}{(C)(D)}$$



LCD = $(B)(C)(D)$

  - Multiply the numerator and denominator of each rational expression by whichever factor of the LCD is missing (outside the circle) from the original denominator.

$$\frac{A}{(B)(C)} + \frac{E}{(C)(D)} = \frac{A(D)}{(B)(C)(D)} + \frac{E(B)}{(C)(D)(B)} = \frac{AD + EB}{BCD}$$

Working Example 1: Add or Subtract Rational Expressions With Common Denominators

Simplify and identify any non-permissible values of the variable.

a) $\dfrac{2x + 4}{x^2 - 9} + \dfrac{7x - 10}{x^2 - 9}$ **b)** $\dfrac{10a + 5}{ab} - \dfrac{3a - 2}{ab}$

Solution

a) Both expressions have the same denominator, so add the numerators.

$$\frac{2x + 4}{x^2 - 9} + \frac{7x - 10}{x^2 - 9} = \frac{(\square) + (\square)}{x^2 - 9}$$

$$= \frac{\square}{x^2 - 9},\ x \neq \underline{\qquad}$$

Add the numerators. The denominator does not change.

b) Both expressions have the same denominator, so subtract the numerators.

$$\frac{10a + 5}{ab} - \frac{3a - 2}{ab} = \frac{(\square) - (\square)}{ab}$$

$$= \frac{\square}{\square},\ a \neq \underline{\qquad},\ b \neq \underline{\qquad}$$

Subtract the numerators. The denominator does not change.

Be careful! You are subtracting −2.

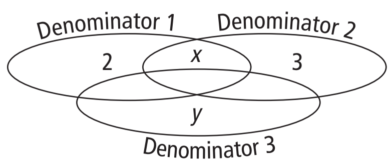
Working Example 2: Determine the Least Common Denominator

Identify the least common denominator for each group of rational expressions.

a) $\dfrac{1}{2x}, \dfrac{1}{3x}, \dfrac{1}{y}$ **b)** $\dfrac{1}{x + 5}, \dfrac{1}{(x + 5)(x - 4)}, \dfrac{1}{(x - 4)(x + 7)}$ **c)** $\dfrac{1}{x^2 + 6x + 9}, \dfrac{1}{x^2 + 8x + 15}$

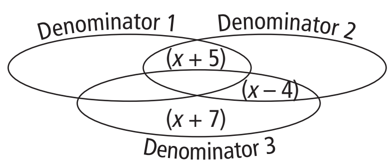
Solution

a) The denominators are ________, ________, and ________.



LCD = $(2)(x)(3)(y) =$ ________

b) $\dfrac{1}{x + 5}, \dfrac{1}{(x + 5)(x - 4)}, \dfrac{1}{(x - 4)(x + 7)}$



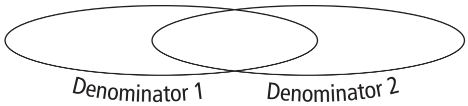
LCD = $(x + 5)($________$)($________$)$

c) $\dfrac{1}{x^2 + 6x + 9}, \dfrac{1}{x^2 + 8x + 15}$

First, factor each denominator. Then, make a diagram.

$x^2 + 6x + 9 =$ (________________)(________________)

$x^2 + 8x + 15 =$ (________________)(________________)



The factor ($x + 3$) appears twice in the first denominator. Make sure it appears twice in the LCD.

LCD = (________________)(________________)(________________)

Working Example 3: Add and Subtract Rational Expressions With Unlike Denominators

Find each sum or difference. Express your answers in simplest form.

a) $\dfrac{x - 15}{x(x + 1)} + \dfrac{x - 4}{x + 1}, x \neq -1, 0$

b) $\dfrac{6x}{x - 5} - \dfrac{240}{x^2 - 2x - 15}, x \neq -3, 5$

Solution

a) $\dfrac{x - 15}{x(x + 1)} + \dfrac{x - 4}{x + 1}$

The two expressions have unlike denominators.

Determine the LCD: ____________________________

The first expression already contains all the factors of the LCD.

The second expression is missing the factor ________________.

Multiply the numerator and denominator of the second expression by the missing factor.

$$\frac{x - 15}{x(x + 1)} + \frac{x - 4}{x + 1} = \frac{x - 15}{x(x + 1)} + \frac{(x - 4)}{(x + 1)} \times \frac{(\boxed{})}{(\boxed{})}$$

$$= \frac{x - 15}{x(x + 1)} + \frac{\boxed{}}{x(x + 1)}$$

Now both expressions have a common denominator.

$$= \frac{(\boxed{}) + (\boxed{})}{x(x + 1)}$$

$$= \frac{\boxed{}}{x(x + 1)}, x \neq -1, 0$$

Simplify the numerator by collecting like terms. If possible, factor the result.
Check that the result cannot be simplified further.

b) The two expressions have unlike denominators. Factor.

$$\frac{6x}{x-5} - \frac{240}{x^2-2x-15} = \frac{6x}{x-5} - \frac{240}{(\square)(\square)}$$

Determine the LCD: ____________________

Multiply the numerator and denominator of the first expression by the missing factor. Simplify the numerator of the first expression.

$$\frac{6x}{x-5} - \frac{240}{x^2-2x-15} = \frac{6x(\square)}{(x-5)(\square)} - \frac{240}{(\square)(\square)}$$

$$= \frac{(\square)}{(x-5)(\square)} - \frac{240}{(\square)(\square)}$$

$$= \frac{(\square) - 240}{(x-5)(\square)}$$

$$= \frac{\square}{\square}, x \neq -3, 5$$

Simplify the numerator by collecting like terms. If possible, factor the result. Check if the result can be further simplified.

Check Your Understanding

Practise

1. Add. Express answers in simplest form. Identify any non-permissible values.

a) $\frac{7x + 1}{4x} + \frac{3x - 4}{4x}$

b) $\frac{x^2 + 1}{x - 8} + \frac{2x + 1}{x - 8}$

2. Subtract the following rational expressions. Express answers in simplest form. Identify any non-permissible values.

a) $\frac{4x - 8}{x^2} - \frac{x + 1}{x^2}$

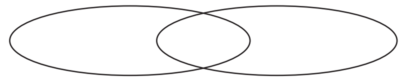
b) $\frac{x^2 + 6x}{x^2 - 25} - \frac{4x - 1}{x^2 - 25}$

c) $\frac{x^2 - 1}{x + 9} - \frac{8 - 8x}{x + 9}$

d) $\frac{x^2 + 3x - 20}{6 - x} - \frac{3x + 16}{6 - x}$

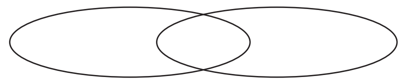
3. Determine the least common denominator (LCD) of the following pairs of rational expressions. Leave your answers in factored form. Be sure to put binomial factors in brackets.

a) $\frac{1}{5x}, \frac{1}{xy}$



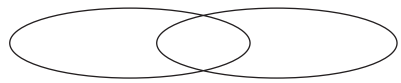
LCD = ____________________________

b) $\frac{1}{3x}, \frac{1}{x^2}$



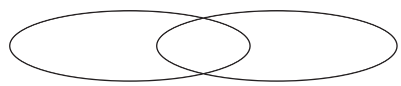
LCD = ____________________________

c) $\frac{1}{x + 1}, \frac{1}{x(x + 1)}$



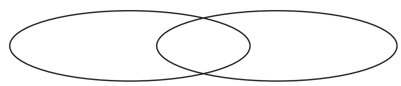
LCD = ____________________________

d) $\frac{1}{x - 7}, \frac{1}{x + 5}$



LCD = ____________________________

e) $\frac{1}{x^2 + 8x + 12}, \frac{1}{x^2 + 4x - 12}$



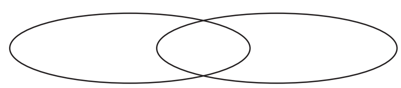
$x^2 + 8x + 12 =$ (________________)(________________)

$x^2 + 4x - 12 =$ (________________)(________________)

LCD = ____________________________

f) $\frac{1}{x^2 - 10x + 25}, \frac{1}{5 - x}$

Remember to look for opposites.



LCD = ____________________________

Also try #4 on page 336 of *Pre-Calculus 11*.

4. Find the sum or difference, as indicated. State any non-permissible values of the variable(s).

a) $\frac{1}{6x} + \frac{1}{xy}$ LCD = ____________________

b) $\frac{1}{7x} - \frac{1}{x^2}$ LCD = ____________________

c) $\frac{1}{x - 3} + \frac{1}{x(x - 3)}$ LCD = ____________________

d) $\frac{1}{x + 7} - \frac{1}{(x - 8)(x + 7)}$ LCD = ____________________

e) $\frac{1}{x^2 + 6x + 8} + \frac{1}{x^2 + x - 12}$ LCD = ____________________

f) $\frac{1}{4 - x} - \frac{1}{x^2 - 8x + 16}$ LCD = ____________________

The factor ($x - 4$) appears twice in the denominator of the second fraction. Make sure it appears twice in the LCD.

5. Determine the sum. Express your answers in simplest form. List the non-permissible values of the variable.

a) $\frac{a}{a+7} + \frac{2}{a+5}$ LCD = ______________________

b) $\frac{6x-19}{x^2-3x-4} + \frac{x-5}{x-4}$ LCD = ______________________

6. Determine the difference. Express your answers in simplest form. List the non-permissible values of the variable.

a) $\frac{3x}{x-9} - \frac{2x}{x-12}$ LCD = ______________________

b) $\frac{2x+2}{x^2+4x-12} - \frac{x+1}{x^2-4}$ LCD = ______________________

Apply

7. At Nathan's Deli in New York City, there is an annual hot-dog eating contest. Contestants have 12 min to consume as many hot dogs as they can. Suppose you can eat an average of n hot dogs per minute.

a) Write an expression for the number of minutes it would take to eat 5 hot dogs.

Remember to state the non-permissible values.

b) Suppose that, after you have eaten 5 hot dogs, your average rate of consumption slows down by 1 hot dog per minute. Write an expression for the number of minutes it would take to eat the next 5 hot dogs.

c) Using the information from above, write a sum of rational expressions (in simplest form) representing the number of minutes it would take to eat 10 hot dogs.

8. Suppose that you are competing in the 10-km race at the Vancouver Sun Run.

a) For the first 8 km, you run at an average rate of y km/h. Write an expression for the number of *minutes* it would take to run 8 km.

b) After 8 km, you get tired. You run 5 km/h slower for the rest of the race. Write an expression for the number of *minutes* it takes to run the final 2 km of the race.

c) Using the information from above, write a sum of rational expressions (in simplest form) representing the number of minutes it takes you to complete the race.

See pages 337–340 of *Pre-Calculus 11* for more application questions.

9. Simplify the following expressions. List any non-permissible values for x.

a) $\frac{x + 1}{x - 2} + \frac{x^2 + 4x - 5}{x^2 + 5x - 14} \times \frac{x^2 + 4x - 21}{x - 1}$

Do multiplication and division before addition and subtraction.

b) $\frac{x^2 - 3x - 18}{x^2 + 10x} \div \frac{x^2 - 13x + 42}{x^2 + 3x - 70} - \frac{x + 5}{2x + 3}$

Connect

10. Describe how to determine the least common denominator (LCD) of two rational expressions. Create an example to help you illustrate your explanation.

6.4 Rational Equations

KEY IDEAS

- A rational equation is an equation that contains at least one rational expression. You can use rational equations to solve problems in which an unknown value is in the denominator. For example, $x + \frac{4}{x} = 4$ is a rational equation.
- To solve a rational equation,
  – factor all denominators (if necessary) and determine the least common denominator (LCD)
  – identify any non-permissible values of the variable (from the LCD)
  – multiply all terms on both sides of the equation by the LCD, and then simplify
  – solve for the variable
  – check that the solution(s) are permissible, and that they make sense in the given context
  – verify that any remaining solutions are correct in the original equation

In the example given above, the LCD is x. The non-permissible value is $x = 0$.

Multiply each term on both sides of the equation by x, and then simplify.

$$x + \frac{4}{x} = 4$$
$$x\left(x + \frac{4}{x}\right) = x(4)$$
$$x(x) + x\left(\frac{4}{x}\right) = 4x$$
$$x^2 + 4 = 4x$$

Collect all terms of the quadratic on the same side of the equation to solve for x.

$$x^2 - 4x + 4 = 0$$
$$(x - 2)(x - 2) = 0$$

Set each factor equal to 0 and solve for x: $x - 2 = 0$

$x = 2$

This value is permitted (recall that $x \neq 0$).

Verify in the original equation.

Left Side	Right Side
$x + \frac{4}{x}$	4
$= \mathbf{2} + \frac{4}{\mathbf{2}}$	
$= 2 + 2$	
$= 4$	

Left Side = Right Side

Therefore, $x = 2$ is a solution.

Working Example 1: Solve a Rational Equation

a) Solve the equation.

$$\frac{x^2 + 25}{x - 7} + \frac{x + 5}{2} = \frac{2x^2 - 12x - 9}{2x - 14}$$

b) Verify your solution(s).

Solution

a) Factor the numerators and denominators.

The least common denominator (LCD) is ______________________.

The non-permissible value is $x =$ ________.

Multiply each term on both sides of the equation by the LCD, and then simplify.

$$(\underline{\qquad\quad})\left(\frac{x^2 + 25}{x - 7}\right) + (\underline{\qquad\quad})\left(\frac{x + 5}{2}\right) = (\underline{\qquad\quad})\left(\frac{2x^2 - 12x - 9}{2(x - 7)}\right)$$

$$(\underline{\qquad\quad})(x^2 + 25) + (\underline{\qquad\quad})(x + 5) = 2x^2 - 12x - 9$$

Collect all terms of the quadratic on the same side of the equation to solve for x.
Set each factor equal to zero and solve for x:

$(\underline{\qquad\qquad\quad}) = 0$ or $(\underline{\qquad\qquad\quad}) = 0$

$x =$ $x =$

b) For $x = -4$:

Left Side	Right Side
$\frac{x^2 + 25}{x - 7} + \frac{x + 5}{2}$	$\frac{2x^2 - 12x - 9}{2x - 14}$
$= \frac{(\mathbf{-4})^2 + 25}{(\mathbf{-4}) - 7} + \frac{(\mathbf{-4}) + 5}{2}$	$= \frac{2(\mathbf{-4})^2 - 12(\mathbf{-4}) - 9}{2(\mathbf{-4}) - 14}$
$= \frac{\square}{\square} + \frac{\square}{\square}$	$= \frac{\square}{\square}$
$= \frac{\square}{\square}$	

Therefore, $x = -4$ ________________ a solution.
(*is* or *is not*)

For $x = -6$:

Left Side	Right Side
$\frac{x^2 + 25}{x - 7} + \frac{x + 5}{2}$	$\frac{2x^2 - 12x - 9}{2x - 14}$
$= \frac{(\mathbf{-6})^2 + 25}{(\mathbf{-6}) - 7} + \frac{(\mathbf{-6}) + 5}{2}$	$= \frac{2(\mathbf{-6})^2 - 12(\mathbf{-6}) - 9}{2(\mathbf{-6}) - 14}$
$= \frac{\square}{\square} + \frac{\square}{\square}$	$= \frac{\square}{\square}$
$= \frac{\square}{\square}$	

Therefore, $x = -6$ ________ a solution.
(*is* or *is not*)

Working Example 2: Solve a Rational Equation With an Extraneous Root

Solve the equation and verify your solution(s) in the original equation.

$$\frac{5x}{x - 4} + 2 = \frac{3x + 8}{x - 4}$$

Solution

The least common denominator (LCD) is ________. The non-permissible value is $x =$ ________.

Multiply each term on both sides of the equation by the LCD. Then, simplify and solve for x.

$$(\_\_\_\_)\left(\frac{5x}{x - 4}\right) + (\_\_\_\_)(2) = (\_\_\_\_)\left(\frac{3x + 8}{x - 4}\right)$$

However, from the original equation, $x \neq 4$.
Therefore, there is no solution to the rational equation.

Working Example 3: Sharing a Task

Andrea and Phary are sharing a bag of popcorn at the movies.

- By himself, Phary can eat the whole bag of popcorn in 20 min.
- Andrea takes 25 min to eat the whole bag.

If they both eat popcorn at their usual rates, how quickly will they eat the popcorn?

Solution

Let x represent the time, in minutes, it takes Andrea and Phary together to eat the popcorn. Organize the information in a table.

	Time to Eat Popcorn (min)	Fraction of Popcorn Eaten in 1 min	Fraction of Popcorn Eaten in x min
Andrea (A)	25	$\frac{1}{25}$	$\frac{x}{25}$
Phary (P)			
Together	x	$\frac{1}{x}$	$x\left(\frac{1}{x}\right) = 1$

(fraction A eats) + (fraction P eats) = total

$$\frac{x}{25} \quad + \quad \frac{\square}{\square} \quad = \quad 1$$

The answer should lie somewhere between half of Andrea's time and half of Phary's time. Is this the case? Explain your answer.

Working Example 4: Create and Solve a Rational Model

A group of friends go on a 3-h bike ride together. They ride 15 km with the wind at their backs, and then 15 km straight into the wind. The wind adds or subtracts 3 km/h from their speed. What is the average speed of the group of friends with no wind?

Solution

Let x represent the average biking speed with no wind, in kilometres per hour.
Then the average speed with the wind at their backs is $(x + 3)$ km/h, and the average speed riding into the wind is $(x - 3)$ km/h.

	Distance, d (km)	Speed, s (km/h)	Time, t $t = \frac{d}{s}$
With wind	15	$x + 3$	$\frac{15}{x + 3}$
Against wind			
Total			3

(time biking with wind) + (time biking against wind) = (total time)

$$\frac{15}{x + 3} \quad + \quad \boxed{} \quad = \quad 3$$

Use the quadratic equation to solve for x.

$a =$ ________, $b =$ ________, $c =$ ________

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Does either solution for x match any of the non-permissible values?
Do both solutions for x make sense in the context of the problem?

Therefore, the average biking speed with no wind is ________ km/h.

Check Your Understanding

Practise

1. Use the least common denominator to eliminate the fractions from each equation. Do not solve.

a) $\frac{7}{4x} + \frac{3}{4x} = 5x - 3$

The least common denominator (LCD) is ______________________.

The non-permissible value is $x =$ ________.

$(\_\_\_\_\_\_)\left(\frac{7}{4x}\right) + (\_\_\_\_\_\_)\left(\frac{3}{4x}\right) = (\_\_\_\_\_\_)\left(\frac{5x - 3}{1}\right)$

b) $\frac{x + 1}{x - 8} + \frac{2x + 1}{x + 2} = \frac{-7}{(x - 8)(x + 2)}$

The LCD is ______________________.

The non-permissible values are ______________.

c) $\frac{21 - 5x}{x - 3} = \frac{x}{x^2 - 3x} - \frac{x - 2}{2x}$

The LCD is ______________________.

The non-permissible values are ______________.

Also try #1 on page 348 of *Pre-Calculus 11*.

2. Solve and verify each rational equation. Identify any non-permissible values.

a) $\frac{x}{3} + \frac{3}{x} = 2$

The LCD is ______________.

The non-permissible value is

______________.

$(\_\_\_\_\_\_)\frac{x}{3} + (\_\_\_\_\_\_)\frac{3}{x}$

$= (\_\_\_\_\_\_)2$

This value is ______________________.
(*permitted* or *not permitted*)

Verify: For $x =$ _______:

Left Side	Right Side
$\frac{x}{3} + \frac{3}{x}$ $= \frac{(\square)}{3} + \frac{3}{(\square)}$	2

Therefore, $x =$ __________ a correct solution.
(*is* or *is not*)

b) $\frac{2x}{x+1} - \frac{5}{x-1} = \frac{x^2 - 17}{x^2 - 1}$

The LCD is ______________.

The non-permissible values are

______________.

These values are ______________________.
(*permitted* or *not permitted*)

Verify first value: For $x =$ _______:

Left Side	Right Side

Therefore, $x =$ __________ a correct solution.
(*is* or *is not*)

Verify second value: For $x =$ _______:

Left Side	Right Side

Therefore, $x =$ _______

____________________ a correct solution.
(*is* or *is not*)

3. Solve and verify each rational equation. Identify any non-permissible values.

a) $\frac{x - 24}{x^2 - 8x} - \frac{5 - x}{x - 8} = \frac{2x + 3}{x}$

The LCD is ______________.

The non-permissible values are

______________.

Verify permitted value: For $x =$ ________:

Left Side	Right Side

The value $x =$ ________ is permitted but the value $x =$ ________ is not permitted.

Therefore, $x =$ ________ ____________________ a correct solution.
(*is* or *is not*)

b) $\frac{4x - 1}{3x} - \frac{x - 7}{x - 5} = \frac{8x - 10}{3x^2 - 15x}$

The LCD is ______________.

The non-permissible values are

______________.

Verify permissible value(s): For $x =$ ________:

Left Side	Right Side

Are any of the values non-permissible?

Therefore, $x =$ ________ ____________________ a correct solution.
(*is* or *is not*)

📖 Also try #2 to #4 on page 348 of *Pre-Calculus 11*.

Apply

4. Rico and Phania shovel the walkway together. Rico, working alone, can complete the task in 10 min. Phania, working alone, can complete the task in 15 min. How long will it take them to complete the task together?

	Time to Shovel (min)	Fraction Shovelled in 1 min	Fraction Shovelled in x min
Rico (R)			
Phania (P)			
Together (R)			

5. Charlie and Rose share a garden. Charlie can weed the garden, working alone, in 90 min. Rose can finish the task in 75 min. If they work together, how long will it take them to weed the garden?

Define your variable and write an equation to model the situation. Use the table to help you organize the information.

6. The distance from Calgary to Red Deer is approximately 140 km. Some friends leave Calgary at noon, drive to Red Deer, have a half-hour break, and then return to Calgary. They arrive at 3:15 p.m. Traffic is busier on the return trip, so their average speed is 10 km/h slower than on the outbound trip. What is the average speed of the round trip?

How much time in total do the friends spend driving? What is this as a decimal?

Is your answer reasonable in the context of the question? How do you know?

7. Marieke and Kate canoe 8 km up a river (against the current) and 8 km back (with the current). The total paddling time is 2.5 h. If the speed of the current is 2.6 km/h, what is Marieke and Kate's average speed in still water? How much time does it take them to paddle upstream (against the current)?

When travelling with the current, will the speed be faster or slower?

8. Two consecutive integers are represented by n and $n + 1$. The sum of the reciprocals of the integers is $-\frac{43}{462}$. Find the integers.

9. Two consecutive even integers are represented by n and $n + 2$.
The sum of the reciprocals of the integers is $\frac{29}{420}$. Find the integers.

See pages 348–351 of *Pre-Calculus 11* for more practice questions that involve modelling situations using rational equations.

Connect

10. You have worked on a number of word problems in this chapter. Summarize the kinds of situations that you can model using rational equations. Compare these situations with the other kinds of models (linear, quadratic, etc.) that you have used previously in this course.

Chapter 6 Review

6.1 Rational Expressions, pages 229–240

1. Simplify the following rational expressions. State any non-permissible values of the variable(s).

a) $\dfrac{x^2 - 10x + 25}{x^2 - 11x + 30}$

b) $\dfrac{3x^2 + 15x + 12}{3x^2 + 12x}$

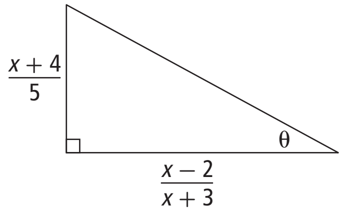
2. Can the expression $\dfrac{-x + 7}{(x - 7)(x + 7)}$ be simplified further? Explain.

6.2 Multiplying and Dividing Rational Expressions, pages 241–251

3. Determine the product. Express your answer in simplest form. State the non-permissible values.

$$\frac{10x^2 - 5x}{x^2 - x - 42} \times \frac{x^2 - 11x + 28}{60x - 15x^2}$$

4. Write an expression for tan θ based on the information in the diagram below. Simplify the expression and state any non-permissible values.



6.3 Adding and Subtracting Rational Expressions, pages 252–262

5. Determine the least common denominator (LCD) for the following set of rational expressions. Leave your answer in factored form.

$$\frac{x + 7}{4x}, \frac{8x}{x^2 - 36}, \frac{1}{x^2 + 6x}$$

6. Determine each difference. Express each answer in simplest form. State the non-permissible values of the variable.

a) $\dfrac{2x^2 - 7x}{x^2 - 100} - \dfrac{x^2 - 2x + 10}{x^2 - 100}$

b) $\dfrac{2x - 3}{x^2 + 5x} - \dfrac{x + 9}{x^2 - 4x - 5}$

6.4 Rational Equations, pages 263–273

7. Emily can shovel the driveway in 25 min. It takes her younger brother Steve 40 min. If they work together to shovel the driveway, how quickly will they finish?

	Time to Shovel (min)	Fraction of Work Done in 1 min	Fraction of Work Done in *t* min
Emily			
Steve			
Together			

Chapter 6 Skills Organizer

Make note of some of the key details and things to remember about the processes you have learned in this unit. Use your class notes, textbook, or questions from this workbook to help you choose examples (or create your own). Some information is provided below to help you get started.

Process	Example	Things to Remember
Simplifying rational expressions		– cancel entire factors only – binomial factors in brackets – watch for opposites $(a - b) = -1(b - a)$
Determining non-permissible values		
Multiplying rational expressions		
Dividing rational expressions		
Finding a common denominator for rational expressions		
Adding rational expressions		
Subtracting rational expressions		
Solving rational equations		
Solving word problems with rational equations		