The Work / Energy Theorem

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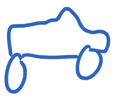
The work/ energy theorem: $W = \Delta E$ [potential, kinetic]

This theorem states that the work done on an object or by an object is equal to the change in energy that the object receives or gives.

As an eqn. $W = \Delta E$

$\Delta E = \Delta E_p + \Delta E_k + \Delta Q$

A car of mass 1000 kg travels at 90 km/h, it is slowed to 36 km/h over a distance of 50 m on a level road. Find a) the work on the car



$m = 1000\ kg$

$V_o = 90\ \frac{km}{hr} \rightarrow 36\ \frac{km}{hr}$ $d = 50 m$ $W = ?$

$$W = \Delta E = \Delta E_k + \Delta E_p$$
$$= \frac{mV_f^2}{2} - \frac{mV_i^2}{2} = \frac{m}{2}(V_f^2 - V_i^2)$$
$$= \frac{1000}{2}\left(10^2 \frac{m}{s} - 25^2 \frac{m}{s}\right)$$
$$= -263{,}000 J = 263 KJ$$

A cat of mass 5.0 kg receives 1.00 x 10 [4] J of work in a kick straight up. If its initial velocity was zero, what velocity will it have at height 20 m?

$V_o = 0$ $V @ 20m = ?$

$$W = \Delta E = \Delta E_k + \Delta E_p$$
$$1 \times 10^4 = \frac{m}{2}(V_f^2 - V_i^2) + mg(h_f - h_i)$$
$$1 \times 10^4 = \frac{5}{2}(V_f^2 - 0) + 5(9.8)(20 - 0)$$
$$V_f = \sqrt{\frac{[1 \times 10^4 - 5(-9.8)(20)](2)}{5}} = 66.3 \frac{m}{s}$$

Other ways to calculate work:

W = F · d USE ONLY WHEN THE FORCE IS CONSTANT!!!!! (Which is almost never)

GRAPH

For a non-constant force use a graph of F vs. d

A cat of mass 5.0 kg is thrown down at 4.0 m/s from height 5.0 m if the velocity is 7.5 m/s when reaching 2.0 m height, how much heat energy was produced?

$$E_{po} + E_{ko} = E_{pf} + E_{kf} + Q$$

$$5(9.8)(5) + \frac{5(4)^2}{2} = 5(9.8)(2) + \frac{5(7.5)^2}{2} + Q$$

$$Q = 46.4\,J$$

What is the force of air resistance in the previous question?

POWER is the rate at which work is done.

A cat of mass 5.0 kg is lifted a height of 2.0m to the input of a renderer in 0.98 s, what Is the power in this experiment?

$$P = \frac{W}{t} = \frac{\Delta E}{t} = \frac{mg(h_f - h_i)}{.98} = \frac{5(9.8)(2-0)}{.98} = 100 \text{ Watts}$$

A cat is dropped from height 12m and strikes the ground at 14 m/s.
If its mass was 2.0 kg, find the expected impact speed without air resistance

$\hookrightarrow V_f = ?$

$E_{p0} + E_{k0} = E_{pf} + E_{kf}$ ~~$+ Q$~~

$(9.8)(12) + 0 = 0 + \frac{V_f^2}{2}$

$V_f = \sqrt{9.8(12)(2)}$
$= 15.3 \ m/s$ → rest is lost to heat.

$E_{p0} + E_{k0} = E_{pf} + E_{kf} + Q$

$(2)(9.8)(12) + 0 = 0 + \frac{(2)14^2}{2} + Q$

$2(9.8)(12) - \frac{2(14^2)}{2} = Q$

$= 39.2 J$

Find the difference between the expected E_k and actual E_k.

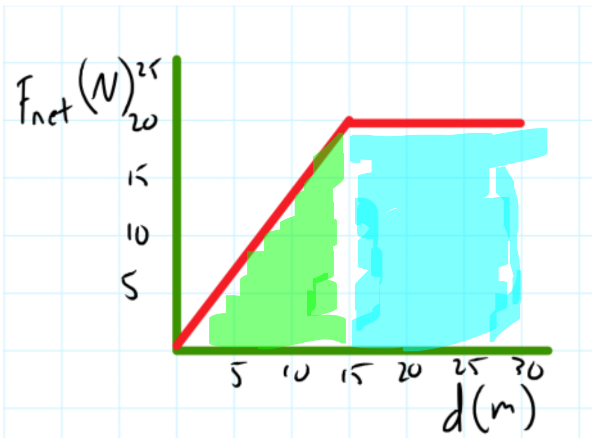
A cat of mass 3.0 kg initially at rest is kicked and viewed at height 2.0 m
with a velocity of 5.0 m/s, if the process took 0.50 s what power was generated
In kicking the cat (ignore air resistance)

$m = 3kg \quad @2m \quad v = 5 \ m/s \quad t = \frac{1}{2} s \quad P = ?$

$$P = \frac{W}{t} = \frac{\Delta E}{t} = \frac{\Delta E_p + \Delta E_k}{t} = \frac{\left[3(9.8)(2-0) + \frac{3(5-0)^2}{2}\right]}{1/2}$$

$= 193 W$

A graph below show the force used to accelerate a 10 kg cat from rest across
A displacement as shown. Find the final velocity of the cat if 100 J of heat energy
is generated in the process.



$$W = \Delta + \square$$
$$= \frac{bh}{2} + bh$$
$$= \frac{15(20)}{2} + 20(15)$$
$$= 450\text{ J}$$

$$V_f = ?$$

$$W = \Delta E = \Delta E_p + \Delta E_k + Q$$
$$450 = 0 + \frac{10v^2}{2} + 100$$
$$v = \sqrt{\frac{(450 - 100)2}{10}} = 8.37\ \text{m/s}$$

A cat of mass 10 kg is dropped from 10 m and strikes the ground at 10 m/s. Find the work
Done by the force of friction (hint how much Q is produced) and find the force of friction.

$$W = \Delta E_p + \Delta E_k$$
$$E_{po} + E_{ko} = E_{pf} + E_{kf} + Q$$
$$10(9.8)(10) + 0 = 0 + \frac{10(10^2)}{2} + Q$$
$$10(9.8)(10) - 5(100) = Q$$
$$480\text{ J} = Q$$

$$W = F \cdot d$$
$$480 = F(10)$$
$$48 = F_f$$

GRAPH FACTS!

1. Graphs are simply a picture which represent an equation
2. You need to be able to recognize the ability to swap between graphs and equations
3. All graphs have an X and Y axis <= identify what is on the axes
4. You will be asked to solve for some other variable
5. Identify an eqn. with the axes variables and the one you are asked to find.
   Manipulate the eqn. to solve for the variable asked to find
   If you have (Y axis / X axis) the variable is the slope
   If you have (Y axis x X axis) the variable is the area