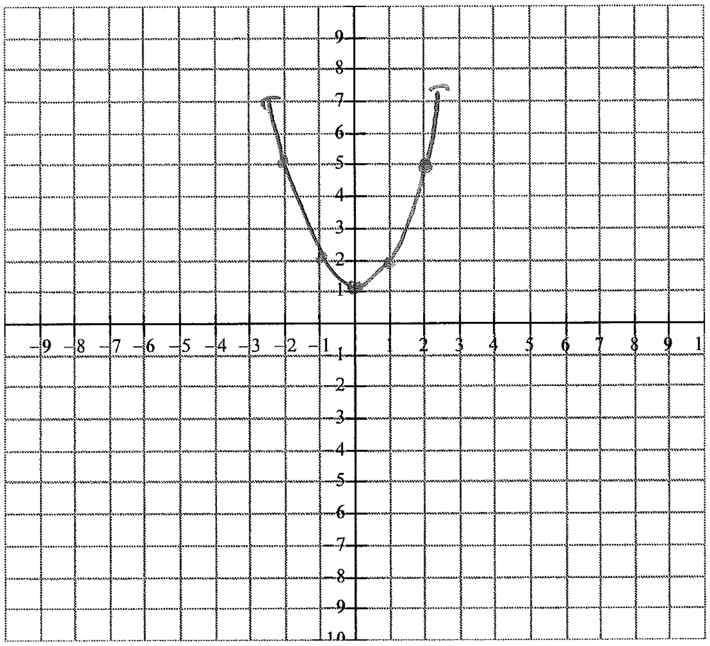
Pre-Calculus 11

Name: Key
Block: ______

Chapter 3
Quadratic Functions Assignment

1. Graph each equation and then answer the given questions.

(a) $y = x^2 + 1$



Vertex: (0,1)

Axis of Symmetry: $x = 0$

Direction of Opening: Up

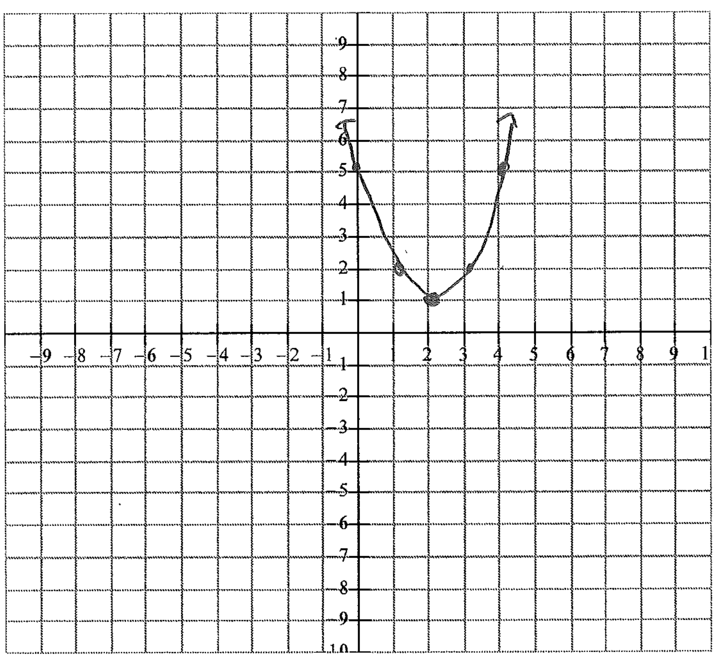
Max or Min: $y = 1$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

Up 1

(b) $y = (x - 2)^2 + 1$



Vertex: (2,1)

Axis of Symmetry: $x = 2$

Direction of Opening: Up

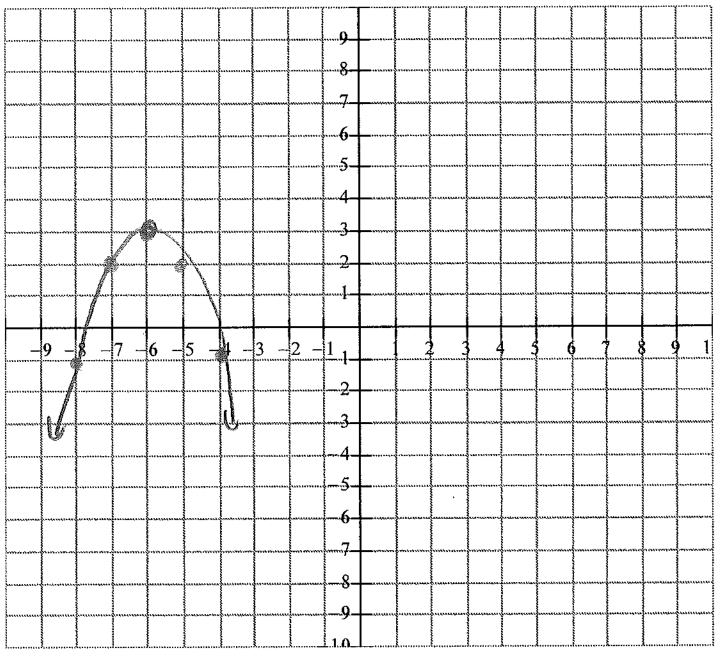
Max or Min: $y = 1$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

Right 2
Up 1

(c) $y = -(x + 6)^2 + 3$



Vertex: (-6,3)

Axis of Symmetry: $x = -6$

Direction of opening: Down

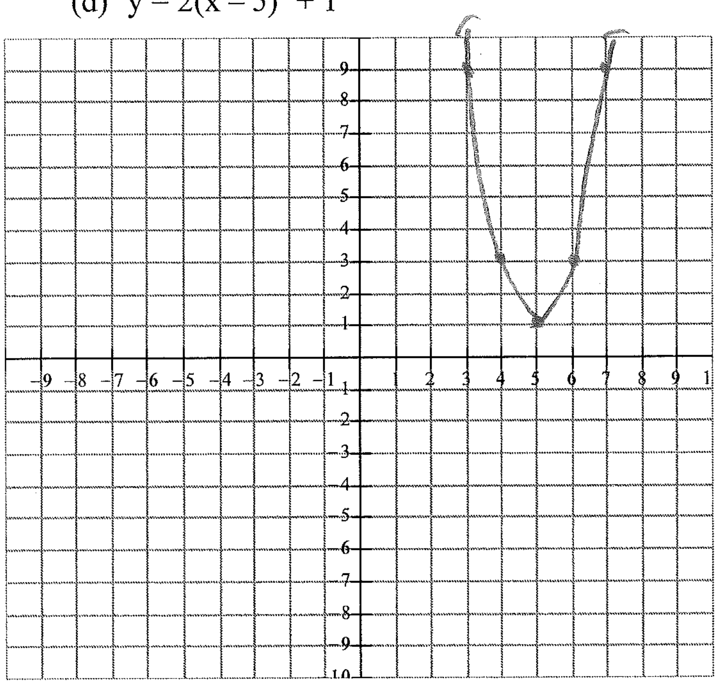
Max or Min: $y = 3$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \leq 3, y \in \mathbb{R}\}$

Opens down
Left 6
Up 3

(d) $y = 2(x - 5)^2 + 1$



Vertex: (5,1)

Axis of symmetry: $x = 5$

Direction of Opening: Up

Max or min: $y = 1$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

Vertical stretch x2
Right 5
Up 1

2. Write the equation of the quadratic in vertex form given the following.

(a) Vertex (0, 3) passing through the point (-1, 2)

p q x y

$y = a(x-p)^2 + q$

$2 = a(-1-0)^2 + 3$

$2 = a(-1)^2 + 3$

$2 = a + 3$

$-1 = a$

$y = -1(x+0)^2 + 3$

$= -1x^2 + 3$

(b) Vertex (-5, -2) passing through the point (-3, 0)

p q x y

$y = a(x-p)^2 + q$

$0 = a(-3+5)^2 - 2$

$0 = a(2)^2 - 2$

$0 = a(4) - 2$

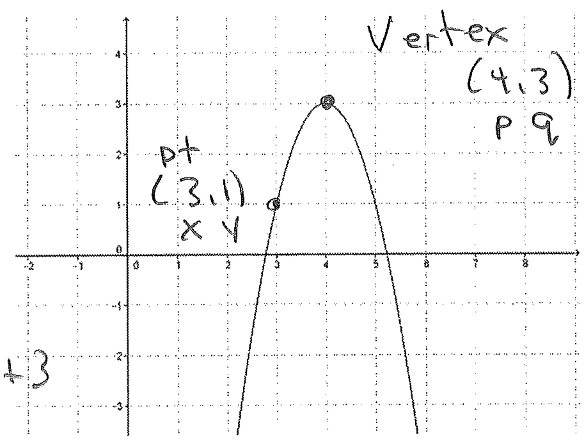
$2 = a(4)$

$\frac{2}{4} = a$

$\frac{1}{2} = a$

$y = \frac{1}{2}(x+5)^2 - 2$

(c) Given the graph



$y = a(x-p)^2 + q$

$1 = a(3-4)^2 + 3$

$1 = a(-1)^2 + 3$

$1 = a + 3$

$-2 = a$

$y = -2(x-4)^2 + 3$

3. Express each equation in vertex form. No decimals allowed.

(a) $y = (x^2 - 6x) + 8$ $\left(-\frac{6}{2}\right)^2$

$y = (x^2 - 6x + 9 - 9) + 8$ $(-3)^2$ 9

$y = (x^2 - 6x + 9) - 9 + 8$

$y = (x-3)^2 - 1$

(b) $y = (2x^2 + 4x) + 7$

$= 2(x^2 + 2x + 1 - 1) + 7$

$= 2(x^2 + 2x + 1) - 2 + 7$

$= 2(x+1)^2 + 5$

$\left(\frac{2}{2}\right)^2$

$(1)^2$

1

(c) $y = (-5x^2 - 6x) - 30$

$= -5\left(x^2 + \frac{6}{5}x + \frac{9}{25} - \frac{9}{25}\right) - 30$

$= -5\left(x^2 + \frac{6}{5}x + \frac{9}{25}\right) + \frac{9}{5} - 30$

$= -5\left(x + \frac{3}{5}\right)^2 - \frac{141}{5}$

$\left(\frac{6}{5} \div 2\right)^2$

$\left(\frac{6}{5} \cdot \frac{1}{2}\right)^2$

$\left(\frac{6}{10}\right)^2$

$\left(\frac{3}{5}\right)^2$

$9/25$

$-5(-9/25)$

$\frac{45}{25}$

$\frac{9}{5}$

4. State the equation of a quadratic in vertex form if the axis of symmetry is x = -2 and the minimum is -1 and passing through the point (1, 2).

Vertex$(-2, -1)$ $P \in (1, 2)$

$y = a(x-p)^2 + q$

$2 = a(1+2)^2 - 1$

$2 = a(3)^2 - 1$

$2 = a(9) - 1$

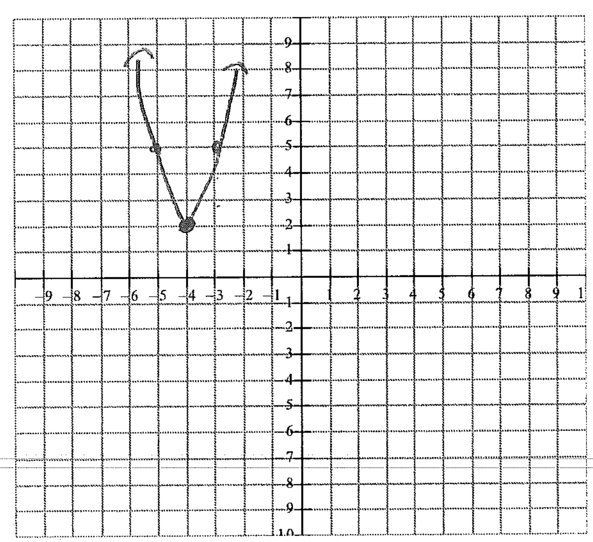
$3 = a(9)$

$\frac{3}{9} = a$

$\frac{1}{3} = a$

$y = \frac{1}{3}(x+2)^2 - 1$

5. Graph using a table of values: $y = 3x^2 + 24x + 50$



x	y
−2	14
−3	5
−4	2 ← vertex
−5	5
−6	14

$x = \frac{-b}{2a}$

$= \frac{-24}{2(3)}$

$= \frac{-24}{6}$

$= -4$ x-value of vertex

$y = 3x^2 + 24x + 50$

$= 3(-2)^2 + 24(-2) + 50$

$= 14$

6. The parabolic path of an aircraft used to simulate weightlessness can be represented by the quadratic equation: $h(t) = -10t^2 + 300t + 9750$ where $h(t)$ is the altitude of the aircraft, in meters, and t is time, in seconds, since weightlessness was achieved. Find:

a) the maximum altitude reached by the aircraft

For a quadratic, max & min values occour at the vertex

$(-10t^2 + 300t) + 9750$

$-10(t^2 - 30t + 225 - 225) + 9750$

$-10(t^2 - 30t + 225) + 2250 + 9750$

$-10(t - 15)^2 + 12000$

Vertex $(15, 12000)$

Max altitude is 12000m

$\left(\frac{-30}{2}\right)^2$

$(-15)^2$

225

b) the number of seconds the aircraft takes to reach its maximum altitude after weightlessness is achieved.

Vertex $(15, 12000)$

15 sec

c) the altitude of the aircraft when weightlessness is first achieved

$t = 0$ $\quad h = -10(0)^2 + 300(0) + 9750$

$h = 9750$ m

d) the number of seconds the simulation of weightlessness lasts if weightlessness is lost at the same altitude as it is achieved.

due to symmetry

15 + 15 = 30 seconds

7. An amusement park charges $8 admission and averages 2000 visitors per day. A survey shows that for each $1 increase in admission cost, 100 fewer people would visit the park.

a) Write an equation to express the revenue, R(x) dollars, in terms of a price increase of x dollars.

Let x = # of increases

price = $8 + 1x$

# of tickets = $2000 - 100x$

R = price × # of tickets

$= (8 + 1x)(2000 - 100x)$

$= 16000 - 800x + 2000x - 100x^2$

$= -100x^2 + 1200x + 16000$

$= -100(x^2 - 12x + 36 - 36) + 16000$

$= -100(x^2 - 12x + 36) + 3600 + 16000$

$= -100(x - 6)^2 + 19600$

$\left(\frac{12}{2}\right)^2$

6^2

36

b) What admission cost gives the maximum revenue?

Look at vertex $(6, 19600)$

$\$19{,}600$ is max revenue

c) What is the maximum revenue?

Vertex $(6, 19600)$

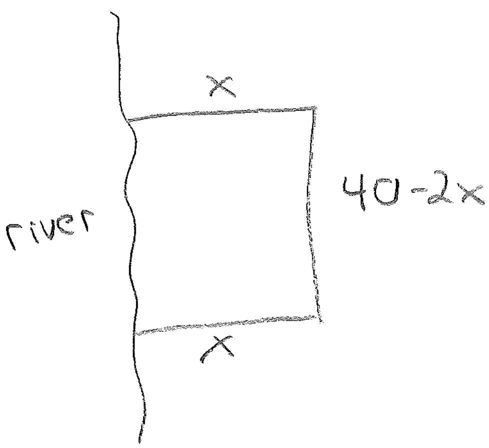
$x \rightarrow$ # of price increases $= 6$

New price $\$8 + 6(\$1) = \$14$

d) How many visitors give the maximum revenue?

$$\begin{aligned}\text{\# of tickets} &= 2000 - 100x \\ &= 2000 - 100(6) \\ &= 2000 - 600 \\ &= 1400 \text{ visitors}\end{aligned}$$

8. A rectangular field is bordered on one side by a river and the other 3 sides by 40 m of fencing. Find the dimensions of the lot and its maximum area.



$$\begin{aligned}A &= L \times W \\ &= (40-2x)x \\ &= 40x - 2x^2 \\ &= (-2x^2 + 40x) \\ &= -2(x^2 - 20x + 100 - 100) \\ &= -2(x^2 - 20x + 100) + 200 \\ &= -2(x-10)^2 + 200\end{aligned}$$

quadratic, need vertex for max

$\left(\frac{-20}{2}\right)^2$

$(-10)^2$

100

Vertex $(10, 200)$

$x = 10$

max area

width $= x = 10$ m

$$\begin{aligned}\text{Length} &= 40 - 2x \\ &= 40 - 2(10) \\ &= 40 - 20 \\ &= 20 \text{ m}\end{aligned}$$