

Radicals Review

Name: Key

Show *all* work to receive full marks.

1. Change to a mixed radical

$$\begin{aligned} \text{a) } & \sqrt{27} \\ & \sqrt{9 \cdot 3} \\ & 3\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } & \sqrt{54x^5y^5} \\ & \sqrt{9 \cdot 6 \cdot x^2 \cdot x^2 \cdot x \cdot y^2 \cdot y^2 \cdot y} \\ & 3x \cdot x \cdot y \cdot y \sqrt{6xy} \\ & 3x^2y^2\sqrt{6xy} \end{aligned}$$

$$\begin{aligned} \text{c) } & 2\sqrt{147} \\ & 2\sqrt{49 \cdot 3} \\ & 2 \cdot 7\sqrt{3} \\ & 14\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{d) } & 4\sqrt[3]{32} \\ & 4\sqrt[3]{8 \cdot 4} \\ & 4 \cdot 2 \cdot \sqrt[3]{4} \\ & 8\sqrt[3]{4} \end{aligned}$$

2. Change to an entire radical

$$\begin{aligned} \text{a) } & 4\sqrt{10} \\ & \sqrt{4^2 \cdot 10} \\ & \sqrt{16 \cdot 10} \\ & \sqrt{160} \end{aligned}$$

$$\begin{aligned} \text{b) } & 4x^3\sqrt{5} \\ & \sqrt[3]{4^3(x^3)^3 \cdot 5} \\ & \sqrt[3]{64x^9 \cdot 5} \\ & \sqrt[3]{320x^9} \end{aligned}$$

$$\begin{aligned} \text{c) } & 2x^3\sqrt{3x} \\ & \sqrt{2^2(x^3)^2 \cdot 3x} \\ & \sqrt{4 \cdot x^6 \cdot 3x} \\ & \sqrt{12x^7} \end{aligned}$$

3. Simplify

$$\begin{aligned} \text{a) } & \sqrt{6} \times \sqrt{8} \\ & \sqrt{6} \times \sqrt{4 \cdot 2} \\ & \sqrt{6} \times 2\sqrt{2} \\ & 2\sqrt{12} \\ & 2\sqrt{4 \cdot 3} \\ & 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } & 5\sqrt{20xy} \times 2\sqrt{5x} \\ & 5\sqrt{4 \cdot 5 \cdot xy} \times 2\sqrt{5x} \\ & 10\sqrt{5xy} \cdot 2\sqrt{5x} \\ & 20\sqrt{25x^2y} \\ & 20 \cdot 5 \times \sqrt{y} \\ & 100x\sqrt{y} \end{aligned}$$

$$\begin{aligned} \text{c) } & -8\sqrt{3} \times 5\sqrt{6} \\ & -40\sqrt{18} \\ & -40\sqrt{9 \cdot 2} \\ & -120\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{d) } & 2\sqrt{24} \times -3\sqrt{18} \\ & 2\sqrt{4 \cdot 6} - 3\sqrt{9 \cdot 2} \\ & 4\sqrt{6} - 9\sqrt{2} \\ & -36\sqrt{12} \\ & -36\sqrt{4 \cdot 3} \\ & -72\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{e) } & 2\sqrt{6} \times 7\sqrt{8} \times 5\sqrt{2} \\ & 2\sqrt{6} \times 7\sqrt{4 \cdot 2} \times 5\sqrt{2} \\ & 2\sqrt{6} \times 14\sqrt{2} \times 5\sqrt{2} \\ & 140\sqrt{24} \\ & 140\sqrt{4 \cdot 6} \\ & 280\sqrt{6} \end{aligned}$$

4. Simplify, leave no radicals in the denominator.

$$\text{a) } \frac{\sqrt{56}}{\sqrt{8}}$$

$$\frac{\sqrt{7}}{\sqrt{7}}$$

$$\text{b) } \frac{\sqrt{48}}{\sqrt{6}}$$

$$\frac{\sqrt{8}}{\sqrt{4 \cdot 2}}$$

$$2\sqrt{2}$$

$$\text{c) } \frac{3\sqrt{20}}{4\sqrt{12}}$$

$$\frac{3\sqrt{4 \cdot 5}}{4\sqrt{4 \cdot 3}}$$

$$\frac{6\sqrt{5}}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{6\sqrt{15}}{24}$$

$$\frac{1\sqrt{15}}{4}$$

$$\text{d) } \frac{3\sqrt{60}}{2\sqrt{27}}$$

$$\frac{3\sqrt{4 \cdot 15}}{2\sqrt{9 \cdot 3}}$$

$$\frac{6\sqrt{15}}{6\sqrt{3}} = \sqrt{5}$$

5. Add/Subtract

$$\text{a) } 5\sqrt{2} - 16\sqrt{2} + 29\sqrt{2}$$

$$18\sqrt{2}$$

$$\text{b) } \sqrt{20} - \sqrt{45}$$

$$\sqrt{4 \cdot 5} - \sqrt{9 \cdot 5}$$

$$2\sqrt{5} - 3\sqrt{5}$$

$$-1\sqrt{5}$$

$$\text{c) } \sqrt{80} + \sqrt{45} - \sqrt{125}$$

$$\sqrt{16 \cdot 5} + \sqrt{9 \cdot 5} - \sqrt{25 \cdot 5}$$

$$4\sqrt{5} + 3\sqrt{5} - 5\sqrt{5}$$

$$2\sqrt{5}$$

$$\text{d) } 5\sqrt{28} - 3\sqrt{63} + 2\sqrt{112}$$

$$5\sqrt{4 \cdot 7} - 3\sqrt{9 \cdot 7} + 2\sqrt{16 \cdot 7}$$

$$10\sqrt{7} - 9\sqrt{7} + 8\sqrt{7}$$

$$9\sqrt{7}$$

$$\text{e) } 2\sqrt[3]{24} + 3\sqrt[3]{81}$$

$$2\sqrt[3]{8 \cdot 3} + 3\sqrt[3]{27 \cdot 3}$$

$$4\sqrt[3]{3} + 9\sqrt[3]{3}$$

$$13\sqrt[3]{3}$$

6. Expand

$$\begin{aligned} \text{a) } & \sqrt{5}(\sqrt{11}-\sqrt{3}) \\ & \sqrt{55}-\sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{b) } & 3\sqrt{3}(\sqrt{3}-2\sqrt{6}) \\ & 3\sqrt{9}-6\sqrt{18} \\ & 9-6\sqrt{9 \cdot 2} \\ & 9-18\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{c) } & (\sqrt{7}-3)(4\sqrt{7}+1) \\ & 4 \cdot 7 + \sqrt{7} - 12\sqrt{7} - 3 \\ & 28 - 11\sqrt{7} - 3 \\ & 25 - 11\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{d) } & (3\sqrt{5}+2\sqrt{3})^2 \\ & (3\sqrt{5}+2\sqrt{3})(3\sqrt{5}+2\sqrt{3}) \\ & 3 \cdot 3\sqrt{25} + 6\sqrt{15} + 6\sqrt{15} + 4\sqrt{9} \\ & 15 + 12\sqrt{15} + 4 \\ & 27 + 12\sqrt{15} \end{aligned}$$

7. Rationalize the denominator.

$$\begin{aligned} \text{a) } & \frac{5}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} \\ & \frac{5\sqrt{7}-5\sqrt{3}}{7+\sqrt{21}-\sqrt{21}-3} \\ & \frac{5\sqrt{7}-5\sqrt{3}}{4} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{3\sqrt{3}+1}{\sqrt{2}+3} \frac{\sqrt{2}-3}{\sqrt{2}-3} \\ & \frac{3\sqrt{6}-9\sqrt{3}+\sqrt{2}-3}{2+3\sqrt{2}-3\sqrt{2}-9} \\ & \frac{3\sqrt{6}-9\sqrt{3}+\sqrt{2}-3}{-7} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{5\sqrt{3}-3\sqrt{5}}{\sqrt{5}-\sqrt{3}} \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\ & \frac{5\sqrt{15}+5\sqrt{3}-3\sqrt{5}-3\sqrt{15}}{5-\sqrt{15}+\sqrt{15}-3} \\ & \frac{2\sqrt{15}}{2} \\ & \sqrt{15} \end{aligned}$$

8. Solve the radical expression.

$$\begin{aligned} \text{a) } & -8 + \sqrt{5a-5} = -3 \\ & \quad \quad \quad +8 \quad \quad \quad +8 \\ & (\sqrt{5a-5})^2 = (5)^2 \quad \text{Check} \\ & 5a-5 = 25 \\ & \quad \quad \quad +5 \quad \quad \quad +5 \\ & 5a = 30 \\ & \quad \quad \quad \frac{5}{5} \quad \quad \quad \frac{30}{5} \\ & a = 6 \end{aligned}$$

$$\begin{aligned} & -8 + \sqrt{5(6)-5} = -3 \\ & -8 + \sqrt{25} = -3 \\ & -8 + 5 = -3 \\ & -3 = -3 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{b) } & \sqrt{7c-54} - c = -6 \\ & \quad \quad \quad +c \quad \quad \quad +c \\ & (\sqrt{7c-54})^2 = (c-6)^2 \quad \text{Check} \\ & 7c-54 = c^2-12c+36 \\ & 0 = c^2-19c+90 \\ & 0 = (c-10)(c-9) \\ & \quad \quad \quad \swarrow \quad \quad \quad \swarrow \\ & c=10 \quad c=9 \end{aligned}$$

$\frac{\text{Check}}{c=10 \checkmark}$
 $c=9 \checkmark$

$$c) (\sqrt{3x+1})^2 = (\sqrt{x+5})^2$$

$$3x+1 = x+5$$

$$\begin{array}{r} -x \quad -x \\ 2x+1 = 5 \end{array}$$

$$\begin{array}{r} -1 \quad -1 \\ 2x = 4 \end{array}$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

Check

$$\sqrt{3(2)+1} = \sqrt{2+5}$$

$$\sqrt{7} = \sqrt{7}$$

$$d) \sqrt{c+7} + 1 = 5 - \sqrt{2c-3}$$

$$\begin{array}{r} -1 \quad -1 \\ (\sqrt{c+7})^2 = (4 - \sqrt{2c-3})^2 \end{array}$$

$$c+7 = 16 - 8\sqrt{2c-3} + 2c-3$$

$$c+7 = 13 + 2c - 8\sqrt{2c-3}$$

$$\begin{array}{r} -2c \quad -13 \quad -13 \quad -2c \\ -1c - 6 = -8\sqrt{2c-3} \end{array}$$

$$\begin{array}{r} -1 \quad -1 \\ -1c - 6 = -8\sqrt{2c-3} \end{array}$$

$$(c+6)^2 = (8\sqrt{2c-3})^2$$

$$c^2 + 12c + 36 = 64(2c-3)$$

$$c^2 + 12c + 36 = 128c - 192$$

$$c^2 - 116c + 228 = 0$$

$$m \ 228 \quad -2, -114$$

$$A = 116$$

$$(c-2)(c-114) = 0$$

$$\begin{array}{r} \swarrow \quad \searrow \\ c=2 \quad c=114 \end{array}$$

Check

$$c=2 \quad \sqrt{2+7} + 1 = 5 - \sqrt{2(2)-3}$$

$$\sqrt{9} + 1 = 5 - \sqrt{1}$$

$$3 + 1 = 5 - 1$$

$$4 = 4 \checkmark$$

$$c=114$$

$$\sqrt{114+7} + 1 = 5 - \sqrt{2(114)-3}$$

$$\sqrt{121} + 1 = 5 - 15$$

$$11 + 1 = -10$$

$$12 = -10 \quad \times$$