

## Chapter 6 Rational Expressions Review

1. State the restrictions on the following rational expressions. Show all work.

a)  $\frac{8x^2y^3}{12y^2}$

$$12y^2 \neq 0$$

$$y^2 \neq 0$$

$$y \neq \sqrt{0}$$

$$y \neq 0$$

b)  $\frac{x-8}{x^2-5x-24}$

$$x^2 - 5x - 24 \neq 0$$

$$(x-8)(x+3) \neq 0$$

$$x-8 \neq 0 \quad x+3 \neq 0$$

$$x \neq 8 \quad x \neq -3$$

c)  $\frac{-5xy}{4x-3y}$

$$4x - 3y \neq 0$$

$$4x \neq 3y$$

$$x \neq \frac{3}{4}y$$

2. Simplify the following rational expressions. Show all work.

a)  $\frac{m^2+5m-6}{m^2+6m}$

$$\frac{(m+6)(m-1)}{m(m+6)}$$

$$\frac{m-1}{m}$$

b)  $\frac{t^2-81}{18t-2t^2}$

$$\frac{(t-9)(t+9)}{2t(9-t)}$$

$$\frac{(t-9)(t+9)}{2t(-1)(t-9)}$$

$$\frac{t+9}{-2t}$$

c)  $\frac{4x^2-1}{2x^2+9x-5}$

$$\frac{(2x-1)(2x+1)}{(x+5)(2x-1)}$$

$$\frac{2x+1}{x+5}$$

$$2x^2 + 9x - 5$$

$$2x^2 + 10x - 1x - 5$$

$$2x(x+5) - 1(x+5)$$

$$(x+5)(2x-1)$$

3. Write a rational expression in expanded form with restrictions on the variable of  $x \neq -3, 5$ . (2 marks)

$$x \neq -3 \quad x \neq 5$$

$$x+3 \neq 0 \quad x-5 \neq 0$$

$$\frac{1}{(x+3)(x-5)}$$

← any polynomial can go on top

3. Write each product in simplest form.

a)  $\frac{5x^2 \cancel{(x-3)}}{\cancel{(x-3)} 10x}$

$$\frac{\overset{1}{\cancel{5x^2}}}{\underset{2}{\cancel{10x}}} = \frac{1x}{2}$$

b)  $\frac{2m-8}{m+3} \times \frac{m^2+4m+3}{m-4}$

$$\frac{2\cancel{(m-4)}}{\cancel{(m+3)}} \cdot \frac{\cancel{(m+3)}(m+1)}{\cancel{(m-4)}} = 2(m+1)$$

4. Write each quotient in simplest form.

a)  $\frac{t-2}{4t-5} \div \frac{4-2t}{4t-5}$

$$\frac{t-2}{4t-5} \cdot \frac{4t-5}{4-2t}$$

$$\frac{\cancel{(t-2)}}{\cancel{(4t-5)}} \cdot \frac{\cancel{(4t-5)}}{-2\cancel{(t-2)}}$$

$$\frac{1}{-2}$$

b)  $\frac{2x^2+3x-2}{x^2+3x-18} \div \frac{6x^2-x-1}{x^2-4x+3}$

$$\frac{2x^2+3x-2}{x^2+3x-18} \cdot \frac{x^2-4x+3}{6x^2-x-1}$$

$$\frac{(2x-1)\cancel{(x+2)}}{\cancel{(x+6)}\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}\cancel{(x-1)}}{\cancel{(2x-1)}(3x+1)}$$

$$\frac{(x+2)(x-1)}{(x+6)(3x+1)}$$

①  $2x^2+3x-2$

$$2x^2+4x-1(x-2) = 2x(x+2)-1(x-2) = (x+2)(2x-1)$$

②  $6x^2-x-1$

$$\frac{6x^2-3x+2x-1}{3x(2x-1)+1(2x-1)} = \frac{(2x-1)(3x+1)}{(2x-1)(3x+1)}$$

5. Add or subtract. Express answers in simplest form.

a)  $\frac{x^2+1}{x-8} + \frac{2x+1}{x-8}$

$$\frac{(x^2+1) + (2x+1)}{x-8}$$

$$\frac{x^2+2x+2}{x-8}$$

b)  $\frac{4n-8}{n^2} - \frac{n+1}{n^2}$

$$\frac{(4n-8) - (n+1)}{n^2}$$

$$\frac{3n-9}{n^2}$$

6. Add or subtract. Express answers in simplest form.

a)  $\frac{6a-19}{a^2-3a-4} + \frac{a-5}{a-4}$

L.C.D  
 $\frac{6a-19}{(a-4)(a+1)} + \frac{(a-5) \cdot (a+1)}{(a-4) \cdot (a+1)}$

$$\frac{6a-19}{(a-4)(a+1)} + \frac{a^2-4a-5}{(a-4)(a+1)}$$

$$\frac{a^2+2a-24}{(a-4)(a+1)}$$

$$\frac{(a-4)(a+6)}{(a-4)(a+1)}$$

$$\frac{a+6}{a+1}$$

b)  $\frac{2x+4}{x^2+8x+12} - \frac{x+1}{x^2-1}$

$$\frac{2(\cancel{x+2})}{(\cancel{x+2})(x+6)} - \frac{(\cancel{x+1})}{(x-1)(\cancel{x+1})}$$

$$\frac{2 \cdot (x-1)}{x+6 \cdot (x-1)} - \frac{1 \cdot (x+6)}{x-1 \cdot (x+6)} \quad \text{L.C.D}$$

$$\frac{2(x-1)}{(x+6)(x-1)} - \frac{(x+6)}{(x+6)(x-1)}$$

$$\frac{2x-2}{(x+6)(x-1)} - \frac{x+6}{(x+6)(x-1)}$$

$$\frac{1x-8}{(x+6)(x-1)}$$

7. Simplify the following.

$$(a) \frac{\left(\frac{y}{2}\right)^6 \left(\frac{3y}{2}\right)^6}{\left(\frac{2y}{3}\right)^6 \left(\frac{4y}{3}\right)^6} \quad \text{L.C.D.} = 6$$

$$\frac{6y + \frac{18y}{2}}{12y - \frac{24y}{3}}$$

$$\frac{6y + 9y}{12y - 8y}$$

$$\frac{15y}{4y}$$

$$\frac{15}{4}$$

$$(b) \frac{4a^2 - 10}{6a^2 - 15} \cdot \frac{2a^2 - 18}{2a^2 - 18}$$

$$\frac{4a^2 - 10}{a - 3} \cdot \frac{2a^2 - 18}{6a^2 - 15}$$

$$\frac{2(2a^2 - 5)}{a - 3} \cdot \frac{2(a^2 - 9)}{3(2a^2 - 5)}$$

$$\frac{2}{a - 3} \cdot \frac{2(a - 3)(a + 3)}{3}$$

$$\frac{4(a + 3)}{3}$$

7. Solve and verify each rational equation. Verify solutions.

$$a) \frac{\left(\frac{x}{3}\right)^{3x} + \left(\frac{3}{x}\right)^{3x}}{\left(\frac{2}{x}\right)^{3x}} \quad \text{L.C.D.} = 3x$$

$$x^2 + 9 = 6x$$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)(x - 3) = 0$$

∴

$$x - 3 = 0$$

$$x = 3$$

Verify

$$\frac{3}{3} + \frac{3}{3} = 2$$

$$1 + 1 = 2$$

$$2 = 2 \quad \checkmark$$

$$b) \frac{x - 24}{x^2 - 8x} - \frac{5 - x}{x - 8} = \frac{2x + 3}{x}$$

$$\frac{x - 24}{x(x - 8)} - \frac{5 - x}{x - 8} = \frac{2x + 3}{x} \quad \text{L.C.D.} = x(x - 8)$$

$$x - 24 - x(5 - x) = (2x + 3)(x - 8)$$

$$x - 24 - 5x + x^2 = 2x^2 - 13x - 24$$

$$x^2 - 4x - 24 = 2x^2 - 13x - 24$$

$$0 = x^2 - 9x$$

$$0 = x(x - 9)$$

$$x = 0 \quad \text{or} \quad x - 9 = 0$$

$$\text{(FAILS)} \quad x = 9$$

Verify

$$\frac{9 - 24}{(9)^2 - 8(9)} - \frac{5 - (9)}{9 - 8} = \frac{2(9) + 3}{9}$$

$$\frac{-15}{9} + 4 = \frac{21}{9}$$

$$\frac{7}{3} = \frac{7}{3} \quad \checkmark$$

8. Mark drives 3 times as fast as Cathy can ride her bike. To travel 50 km, Cathy takes 1 hr more than Mark. How fast is Cathy riding her bike?

Let  $x = \text{Cathy's speed}$

$\Rightarrow 3x = \text{Mark's speed}$

	distance	speed	time
Cathy	50	$x$	$\frac{50}{x}$
Mark	50	$3x$	$\frac{50}{3x}$

time =  $\frac{\text{distance}}{\text{speed}}$

Why -1?  
b/c Cathy takes  
1 hr longer for  
the trip

$$\left(\frac{50}{x}\right)^{3x} - 1 = \left(\frac{50}{3x}\right)^{3x}$$

$$\text{L.C.M} = 3x$$

$$\begin{array}{r} 150 - 3x = 50 \\ -150 \quad \quad -150 \end{array}$$

$$-3x = -100$$

$$x = \frac{100}{3}$$

$$x = 33.3$$

Cathy's speed was approx 33.3 km/hr.