Pre-Calculus 12

Name: Key
Block:

Chapter 8
Logarithmic Functions Assignment

1. a) Write $2^5 = 32$ in logarithmic form.

$5 = \log_2 32$

b) Write $\log_3 m = n$ in exponential form.

$m = 3^n$

2. Use the definition of a logarithm to evaluate $\log_3 81$. Show all work.

$\log_3 81 = 4$

$81 = 3^4$

$(3)^4 = 3^x$

$4 = x$

3. Determine the value of x in each.. Show all work.

a) $\log_5 x = 3$

$x = 5^3$

$x = 125$

b) $\log_x 8 = \frac{3}{4}$

$8 = x^{3/4}$

$8^{4/3} = (x^{3/4})^{4/3}$

$(\sqrt[3]{8})^4 = x$

$2^4 = x$

$16 = x$

4. Rewrite each expression as a single logarithm.

a) $\log_3 x^2 + 3\log_3 x - \log_3 x$

$\log_3 x^2 + \log_3 x^3 - \log_3 x$

$\log_3 (x^2 \cdot x^3) - \log_3 x$

$\log_3 x^5 - \log_3 x$

$\log_3 \frac{x^5}{x}$

$\log_3 x^4$

b) $\log x - 3\log y + \frac{2}{3}\log z$

$\log x - \log y^3 + \log z^{2/3}$

$\log \frac{x}{y^3} + \log z^{2/3}$

$\log \frac{x}{y^3}(z)^{2/3}$

$\log \frac{xz^{2/3}}{y^3}$

5. Use the laws of logarithms to simplify to a single log and then evaluate each expression.

a) $\log_6 3 + \log_6 12$

$\log_6 (3 \cdot 12)$

$\log_6 36$

2

b) $2\log_2 12 - (\log_2 6 + \frac{1}{3}\log_2 27)$

$\log_2 12^2 - (\log_2 6 + \log_2 27^{1/3})$

$\log_2 144 - (\log_2 6 + \log_2 \sqrt[3]{27})$

$\log_2 144 - (\log_2 6 + \log_2 3)$

$\log_2 144 - \log_2 (6 \cdot 3)$

$\log_2 144 - \log_2 18$

$\log_2 \frac{144}{18}$

$\log_2 8 = 3$

6. Write each expression as a single logarithm in simplest form.

a) $2\log x + 3\log\sqrt{x} - \log x^3$

$\log x^2 + 3\log x^{1/2} - \log x^3$

$\log x^2 + \log (x^{1/2})^3 - \log x^3$

$\log x^2 + \log x^{3/2} - \log x^3$

$\log (x^2 \cdot x^{3/2}) - \log x^3$

$\log x^{7/2} - \log x^3$

$\log \frac{x^{7/2}}{x^3}$

$\log x^{1/2}$

$2+\frac{3}{2}$ $\quad \frac{7}{2}-3$

$\frac{4}{2}+\frac{3}{2}$ $\quad \frac{7}{2}-\frac{6}{2}$

$\frac{7}{2}$ $\quad \frac{1}{2}$

b) $\log(x^2 - 25) - 2\log(x + 5)$

$\log \frac{x^2-25}{x+5}$

$\log \frac{(x-5)\cancel{(x+5)}}{\cancel{x+5}}$

$\log (x-5)$

6. Use the laws of logarithms to isolate x in the expression $\log_5 25x = 3$.

$\log_5 25x = 3$

$25x = 5^3$

$25x = 125$

$x = 5$

7. State the transformations, in order of application, to transform $y = \log_c x$ to $y = 3\log_5(4(x-2)) + 6$.

1. Vertical stretch $\times 3$
2. Horizontal stretch $\times \frac{1}{4}$
3. Right 2
4. Up 6

8. Write the equations that correspond to the following transformations of $y = \log_5 x$

a) vertically stretched by a factor of 2 and translated 3 units to the left

$y = 2\log_5 (x+3)$

b) reflected on the x-axis, stretched horizontally by a factor or ½ , translated 3 units to the right and 4 units up

$y = -\log_5 [4(x-3)] + 4$

9. For the equation $y = 3\log_5(6(x-2)) - 4$, state:

a) domain

$6(x-2) > 0$

$x-2 > 0$

$x > 2$

$\{x \mid x > 2, x \in \mathbb{R}\}$

b) range

$\{y \mid y \in \mathbb{R}\}$

c) equation of the asymptote

$x = 2$ (horizontal translation)

d) x-intercept (if it exists)

$y = 0$

$0 = 3\log_5(6(x-2)) - 4$

$4 = 3\log_5(6(x-2))$

$\frac{4}{3} = \log_5(6(x-2))$

$5^{4/3} = 6(x-2)$

$5^{4/3} = 6x - 12$

$5^{4/3} + 12 = 6x$

$\frac{5^{4/3} + 12}{6} = x$

$3.42 \approx x$

e) y-intercept (if it exists)

$x = 0$

$y = 3\log_5(6(0-2)) - 4$

$y = 3\log_5(-12) - 4$

Not possible, no y-intercept

10. Sketch the graph of $y = -\log_4(x+1) - 8$. Show your work (chart) and do not use a graphing calculator.

① $y = \log_4 x$

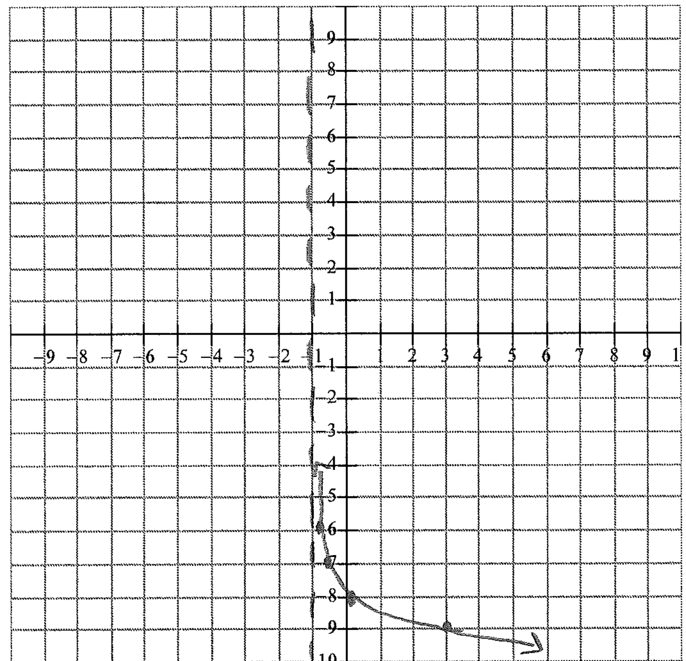
x	y
1/16	-2
1/4	-1
1	0
4	1
16	2

powers of 4

② $y = -\log_4 x$

x	y
1/16	2
1/4	1
1	0
4	-1
16	-2

Reflect over x-axis



vertical asymptote

③ $y = -\log_4(x+1) - 8$

x	y
-15/16	-6
-3/4	-7
0	-8
3	-9
15	-10

Left 1
Down 8

Domain: $\{x \mid x > -1,\ x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

Asymptote(s): $x = -1$

11. Solve. Check for extraneous roots.

a) $\log_6(x-3)+\log_6(x+6)=2$

$\log_6 (x-3)(x+6) = 2$

$(x-3)(x+6) = 6^2$

$x^2+3x-18 = 36$

$x^2+3x-54=0$

$(x+9)(x-6) = 0$

$x=-9 \qquad x=6$

$\underline{x=-9}$

$\log_6(-9-3) + \log_6(-9+6) = 2$

$\log_6 -12 + \log_6 -3 = 2$

Fails

$\underline{x=6}$ $\log_6(6-3)+\log_6(6+6)=2$

$\log_6 3+\log_6 12=2$

$\log_6 36 = 2$

$2=2$ ✓

b) $\log x + \log(x-1) = \log(4x)$

$\log x(x-1) = \log(4x)$

$x(x-1) = 4x$

$x^2 - x = 4x$

$x^2-5x=0$

$x(x-5)=0$

$x=0 \qquad x=5$

$\underline{x=0}$ $\log 0 + \log(0-1) = \log 4(0)$

Fails

$\underline{x=5}$ $\log 5+\log(5-1) = \log 5(4)$

$\log 5+\log 4 = \log 5 + \log 4$ ✓

12. Solve. Express your answer as an exact value (with logs) and as a decimal value correct to the nearest hundredth.

a) $3^{2x+1} = 75$

$2x+1 = \log_3 75$

$2x = \log_3 75 - 1$

$2x = \frac{\log 75}{\log 3} - 1$

$x = \dfrac{\frac{\log 75}{\log 3} - 1}{2}$

$x \approx 1.46$

b) $2^{2x-5} = 6^{x+2}$

$\log 2^{2x-5} = \log 6^{x+2}$

$(2x-5)\log 2 = (x+2)\log 6$

$2x\log 2 - 5\log 2 = x\log 6 + 2\log 6$

$2x\log 2 - x\log 6 = 5\log 2 + 2\log 6$

$x(2\log 2 - \log 6) = 5\log 2 + 2\log 6$

$x = \dfrac{5\log 2 + 2\log 6}{2\log 2 - \log 6}$

$x \approx -17.39$

13. A water filter removes 40% of the impurities in a sample of water.

a) Write an exponential equation to determine the percent of impurities remaining, P, after the water has passed through n filters.

$A = A_0 \, c^{t/T}$ $\qquad A_0 = 100\%$

$P = 1\,(0.6)^{n}$ $\qquad C = 100 - 40 = 60\%$

$T = 1$

b) What percent of impurities will remain after the water has passed through 3 filters?

$P = 1\,(0.6)^3$

$P = 0.216$

$\Rightarrow 21.6\%$

c) How many filters are needed to remove at least 99% of impurities in the water?

$0.01 = (0.6)^n$

$\log 0.01 = \log(0.6)^n$

$\log 0.01 = n \log 0.6$

$\dfrac{\log 0.01}{\log 0.6} = n$

$\rightarrow 9.02 = n$

$\Rightarrow 10$ filters.

14. According to Kleiber's law, a mammal's resting metabolic rate, R, in kilocalories per day, is related to its mass, m, in kilograms, by the equation $\log R = \log 73.3 + 0.75 \log m$. Predict the mass of a wolf with a resting metabolic rate of 1050 kCal/day. Answer to the nearest kilogram.

$\log R = \log 73.3 + 0.75 \log m$

$\log 1050 = \log 73.3 + 0.75 \log m$

$\log 1050 - \log 73.3 = 0.75 \log m$

$\dfrac{\log 1050 - \log 73.3}{0.75} = \log m$

$1.54 = \log m$

$10^{1.54} = m$

$34.8 = m$

wolf's mass is approx 34.8 kg